New Measurement of the Electron Magnetic Moment and the Fine Structure Constant

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Almost finished student: David Hanneke
Earlier contributions: Brian Odom, Brian D’Urso, Steve Peil, Dafna Enzer, Kamal Abdullah, Ching-hua Tseng, Joseph Tan

20 years
6.5 theses

N$F$ 0.1 $\mu$m
Why Does it take Twenty Years and 6.5 Theses?

Explanation 1: We do experiments much too slowly

Explanation 2: Takes time to develop new methods for measurement with 7.6 parts in $10^{13}$ uncertainty

- One-electron quantum cyclotron
- Resolve lowest cyclotron and spin states
- Quantum jump spectroscopy
- Cavity-controlled spontaneous emission
- Radiation field controlled by cylindrical trap cavity
- Cooling away of blackbody photons
- Synchronized electrons probe cavity radiation modes
- Elimination of nuclear paramagnetism
- One-particle self-excited oscillator
New g and $\alpha$

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Magnetic moments, motivation and results

Before this measurement – situation unchanged since 1987

Quantum Cyclotron and Many New Methods $\rightarrow$ electron g
i.e. Why it took $\sim 20$ years to measure g to $7.7$ parts in $10^{13}$

Determining the Fine Structure Constant

Spin-off measurements
• million-fold improved antiproton magnetic moment
• best lepton CPT test (comparing g for electron and positron)
• direct measurement of the proton-to-electron mass ratio

N. Guise (poster)
Also Quint, et al.

NSF
Magnetic Moments, Motivation and Results
Magnetic Moments

magnetic moment  \( \vec{\mu} = g \mu_B \frac{\vec{L}}{\hbar} \)

angular momentum

Bohr magneton \( \frac{e\hbar}{2m} \)

e.g. What is \( g \) for identical charge and mass distributions?

\[
\mu = I A = \frac{e}{2\pi \rho} \left( \frac{\pi \rho^2}{\nu} \right) = \frac{ev\rho}{2} \frac{L}{mv\rho} = \frac{e}{2m} L = \frac{e\hbar}{2m} \frac{L}{\hbar}
\]

\( \Rightarrow \quad g = 1 \)
Magnetic Moments

\[ \vec{\mu} = g \mu_B \frac{\vec{J}}{\hbar} \]

angular momentum

Bohr magneton \( \frac{e\hbar}{2m} \)

\( g = 1 \) cyclotron motion, identical charge and mass distribution

\( g = 2 \) spin for simplest Dirac particle

\( g = 2.002\ 319\ 304 \ldots \) simplest Dirac spin, plus QED

(if electron g is different the electron must have substructure)
Previous Status: **Electron** Magnetic Moment

UW, 1987

\[
\begin{align*}
\text{Electron (UW)}: \quad g/2 & = 1.0011596521884 \pm 0.00000000043 \quad 4.3 \text{ ppt} \\
\text{Positron (UW)}: \quad g/2 & = 1.0011596521879 \pm 0.00000000043 \quad 4.3 \text{ ppt}
\end{align*}
\]

**Electron magnetic moment**

- test QED (with another measured fine structure constant)
- measure fine structure constant (with QED theory)
- best lepton CPT test by comparing electron and positron
Previous Status: **Electron** and **Muon** Moments

**UW, 1987**  
**E821, 2004**

*Electron (UW)*: \( g / 2 = 1.0011596521884 \pm 0.0000000000043 \) 4.3 ppt

*Positron (UW)*: \( g / 2 = 1.0011596521879 \pm 0.0000000000043 \) 4.3 ppt

*Muon \( \mu^+ \) (E821)*: \( g / 2 = 1.0011659203 \pm 0.0000000008 \) 800 ppt

*Muon \( \mu^- \) (E821)*: \( g / 2 = 1.0011659214 \pm 0.0000000008 \) 800 ppt

**Electron magnetic moment**

- test QED (with another measured fine structure constant)
- measure fine structure constant (with QED theory)
- best lepton CPT test by comparing electron and positron

**Muon magnetic moment**

- look for unexpected new physics in expected forms

**Electron more accurate by 200 (\( \rightarrow 1000 \))**

Muon more sensitive to “unexpected” new physics
New Measurement of Electron Magnetic Moment

\[ \vec{\mu} = g \mu_B \frac{\vec{S}}{\hbar} \]

Bohr magneton \( \frac{e\hbar}{2m} \)

\[ g/2 = 1.00115965218085 \pm 0.0000000000076 \times 10^{-13} \]

- First improved measurement since 1987
- Nearly six times smaller uncertainty
- 1.7 standard deviation shift
- Likely more accuracy coming
- 1000 times smaller uncertainty than muon g

Why Measure the Electron Magnetic Moment?

1. The electron g-value is a basic property of the simplest of elementary particles
2. Use measured g and QED to extract fine structure constant (Will be even more important when we change mass standards)
3. Wait for another accurate measurement of $\alpha \rightarrow$ test QED
4. Best lepton CPT test $\rightarrow$ compare g for electron and positron
5. Look for new physics
   • Is g given by Dirac + QED? If not $\rightarrow$ electron substructure
   • Enable the muon g search for new physics
     - provide $\alpha$
     - test of QED
     Needed so the large QED term can be subtracted to see if there is new physics – heavy particles, etc.
QED Relates Measured $g$ and Measured $\alpha$

$$\frac{g}{2} = 1 + C_1 \left( \frac{\alpha}{\pi} \right) + C_2 \left( \frac{\alpha}{\pi} \right)^2 + C_3 \left( \frac{\alpha}{\pi} \right)^3 + C_4 \left( \frac{\alpha}{\pi} \right)^4 + \ldots \delta\alpha$$

1. Use measured $g$ and QED to extract fine structure constant
2. Wait for another accurate measurement of $\alpha \rightarrow$ Test QED

Kinoshita, Nio, Remiddi, Laporta, etc.

Sensitivity to other physics (weak, strong, new) is low

Measure

QED Calculation

weak/strong
After dinner at the Gabrielse apartment in St. Genis in 2004

\[ \frac{g}{2} = 1 + C_1 \left( \frac{\alpha}{\pi} \right) \]
\[ + C_2 \left( \frac{\alpha}{\pi} \right)^2 \]
\[ + C_3 \left( \frac{\alpha}{\pi} \right)^3 \]
\[ + C_4 \left( \frac{\alpha}{\pi} \right)^4 \]
\[ + \ldots \delta a \]

Basking in the reflected glow of theorists
New Determination of the Fine Structure Constant

\[ \alpha = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\hbar c} \]

- Strength of the electromagnetic interaction
- Important component of our system of fundamental constants
- Increased importance for new mass standard

\[ \alpha^{-1} = 137.035\ 999\ 710 \pm 0.000\ 000\ 096 \quad 7.0 \times 10^{-10} \]

- First lower uncertainty since 1987
- Ten times more accurate than atom-recoil methods

Next Most Accurate Way to Determine $\alpha$ (use Cs example)

Combination of measured Rydberg, mass ratios, and atom recoil

$$\alpha \equiv \frac{1}{4\pi\varepsilon_0} \frac{e^2}{hc}$$

$$\alpha^2 = \frac{2R_\infty}{c} \frac{h}{m_e}$$

$$\alpha^2 = 4R_\infty c \frac{f_{recoil} M_{Cs} M_{12C}}{(f_{D1})^2 M_{12C} m_e}$$

$$R_\infty \equiv \frac{1}{(4\pi\varepsilon_0)^2} \frac{e^4 m_e c}{2h^3 c^2}$$

$$\frac{h}{M_{Cs}} = 2c^2 \frac{f_{recoil}}{(f_{D1})^2}$$

- Now this method is 10 times less accurate
- We hope that will improve in the future $\rightarrow$ test QED

(Rb measurement is similar except get $h/M$[Rb] a bit differently)
Famous Earlier Measurements
No Longer Fit on the Same Scale

Gabrielse
Test of QED

Most stringent test of QED: Comparing the measured electron $g$ to the $g$ calculated from QED using an independent $\alpha$

$$\delta g < 15 \times 10^{-12}$$

• None of the uncertainty comes from $g$ and QED
• All uncertainty comes from $\alpha$[Rb] and $\alpha$[Cs]
• With a better independent $\alpha$ could do a ten times better test
Dear Jerry,

... I love your way of doing experiments, and I am happy to congratulate you for this latest triumph. Thank you for sending the two papers.

Your statement, that QED is tested far more stringently than its inventors could ever have envisioned, is correct. As one of the inventors, I remember that we thought of QED in 1949 as a temporary and jerry-built structure, with mathematical inconsistencies and renormalized infinities swept under the rug. We did not expect it to last more than ten years before some more solidly built theory would replace it. We expected and hoped that some new experiments would reveal discrepancies that would point the way to a better theory. And now, 57 years have gone by and that ramshackle structure still stands. The theorists … have kept pace with your experiments, pushing their calculations to higher accuracy than we ever imagined. And you still did not find the discrepancy that we hoped for. To me it remains perpetually amazing that Nature dances to the tune that we scribbled so carelessly 57 years ago. And it is amazing that you can measure her dance to one part per trillion and find her still following our beat.

With congratulations and good wishes for more such beautiful experiments, yours ever, Freeman.
Test for Electron Substructure

(from any difference between experiment and theory $g$)

$$m^* > \frac{m}{\sqrt{\delta g / 2}} = 130 \text{ GeV} / c^2$$

limited by the independent $\alpha$ values

$$m^* > \frac{m}{\sqrt{\delta g / 2}} = 600 \text{ GeV} / c^2$$

if our $g$ uncertainty was the only limit

Brodsky and Drell, 1980

Not bad for an experiment done at 100 mK, but LEP does better

$$m^* > 10.3 \text{ TeV}$$

LEP contact interaction limit
The New Measurement
The 1987 measurement is already very accurate

\[ 4.3 \text{ ppt} = 4.3 \times 10^{-12} \]

**How Does One Measure \( g \) More Accurately?**

**New Methods**
- One-electron quantum cyclotron
- Resolve lowest cyclotron and spin states
- Quantum jump spectroscopy
- Cavity-controlled spontaneous emission
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→ Give introduction to some new and novel methods
Quantum Cyclotron

one electron $\rightarrow$ quantum
average quantum number $< 1$, etc. $\rightarrow$ quantum

$\nu_c \approx 150$ GHz

$B \approx 6$ Tesla

To realize a quantum cyclotron:
- need cyclotron temperature $<< 7.2$ kelvin
- need sensitivity to detect a one quantum excitation

$\hbar \omega_c = 7.2$ kelvin

$\leftarrow 100$ mK apparatus
First Penning Trap Below 4 K $\rightarrow$ 70 mK

Cyclotron motion comes into thermal equilibrium with its container

$$0.070 \ K \ll 7.2 \ K = \frac{\hbar \omega_c}{k}$$
Particle “Thermometer”

70 mK, lowest storage energy for any charged elementary particles
Lowest Energy Cyclotron Eigenstates States

(ignore magnetron motion in a trap)
For Fun: Coherent State

Eigenfunction of the lowering operator:

\[ a |\alpha\rangle = \alpha |\alpha\rangle \]

Fock states do not oscillate

Coherent state with \( \bar{n} = 1 \)

\[ |\psi\rangle = e^{-\bar{n}/2} \sum_{n=0}^{\infty} \frac{\sqrt{n} e^{i\beta}}{\sqrt{n!}} e^{-i\omega t} |n\rangle, \]

\[ n=0 \quad |\psi\rangle = |0\rangle \]

\[ n = 1 \quad |\psi\rangle = |1\rangle \]

0.1 \( \mu \)m
We Use Only the Lowest Cyclotron States

\[ B \approx 6 \text{ Tesla} \]
\[ \nu_c \approx 150 \text{ GHz} \]
Spin → Two Cyclotron Ladders of Energy Levels

Cyclotron frequency:

\[ \nu_c = \frac{eB}{m} \]

\[
\begin{align*}
\text{n} = 4 & \quad \nu_c \\
\text{n} = 3 & \quad \nu_c \\
\text{n} = 2 & \quad \nu_c \\
\text{n} = 1 & \quad \nu_c \\
\text{n} = 0 & \quad \nu_c \\
\end{align*}
\]

Spin frequency:

\[ \nu_s = \frac{g}{2} \nu_c \]

\[
\begin{align*}
\text{n} = 4 & \quad \nu_c \\
\text{n} = 3 & \quad \nu_c \\
\text{n} = 2 & \quad \nu_c \\
\text{n} = 1 & \quad \nu_c \\
\text{n} = 0 & \quad \nu_c \\
\end{align*}
\]

\[ m_s = -\frac{1}{2} \quad m_s = \frac{1}{2} \]
**Basic Idea of the Fully-Quantum Measurement**

- **Cyclotron frequency:**
  \[ \nu_c = \frac{eB}{m} \]

- **Spin frequency:**
  \[ \nu_s = \frac{g}{2} \nu_c \]

Measure a ratio of frequencies:

\[ \frac{g}{2} \nu_c = \frac{\nu_s}{\nu_c} = 1 + \frac{\nu_s - \nu_c}{\nu_c} \]

B in free space \( \sim 10^{-3} \)

- almost nothing can be measured better than a frequency
- the magnetic field cancels out (self-magnetometer)
Modifications to the Basic Idea

Special relativity shifts the levels

We add an electrostatic quadrupole potential \( V \sim 2z^2 - x^2 - y^2 \) to weakly confine the electron for measurement

\( \rightarrow \) shifts frequencies

The electrostatic quadrupole potential is imperfect

\( \rightarrow \) additional frequency shifts
Modification 1: Special Relativity Shift $\delta$

Cyclotron frequency:
$$v_c = \frac{eB}{m}$$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$v_c - \delta/2$</th>
<th>Cyclotron frequency:</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$v_c - 7\delta/2$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$v_c - 5\delta/2$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$v_c - 3\delta/2$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$v_c - \delta/2$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$v_c$</td>
<td></td>
</tr>
</tbody>
</table>

Spin frequency:
$$v_s = \frac{g}{2} v_c$$

$$m_s = -1/2 \quad m_s = 1/2$$

Not a huge relativistic shift, but important at our accuracy
$$\frac{\delta}{v_c} = \frac{h}{mc^2} \approx 10^{-9}$$

Solution: Simply correct for $\delta$ if we fully resolve the levels
(superposition of cyclotron levels would be a big problem)
Modification 2: Add Electrostatic Quadrupole

\[ V \sim 2z^2 - x^2 - y^2 \]

- Electrostatic quadrupole potential \( \rightarrow \) good near trap center
- Control the radiation field \( \rightarrow \) inhibit spontaneous emission by 200x
Frequencies Shift

Perfect Electrostatic Quadrupole Trap

\[ \nu_c = \frac{eB}{m} \]

\[ \nu_s = \frac{g}{2} \nu_c \]

Imperfect Trap

- tilted B
- harmonic distortions to V

Problem: \( \frac{g}{2} = \frac{\nu_s}{\nu_c} \) not a measurable eigenfrequency in an imperfect Penning trap

Solution: Brown-Gabrielse invariance theorem

\[ \nu_c = \sqrt{(\nu'_c)^2 + (\nu_z)^2 + (\nu_m)^2} \]
Spectroscopy in an Imperfect Trap

- one electron in a Penning trap
- lowest cyclotron and spin states

\[
\frac{g}{2} = \frac{v_s}{v_c} = \frac{\bar{v}_c + (v_s - \bar{v}_c)}{v_c} = \frac{\bar{v}_c + \bar{v}_a}{v_c}
\]

\[
\frac{g}{2} \approx 1 + \frac{\bar{v}_a - (\bar{v}_z)^2}{2\bar{v}_c} + \frac{3\delta}{2} + \frac{(\bar{v}_z)^2}{2\bar{v}_c}
\]

To deduce \( g \) \( \rightarrow \) measure only three eigenfrequencies of the imperfect trap
Detecting One Quantum Cyclotron Transitions
Detecting the **Cyclotron Motion**

- **Cyclotron frequency** \( \nu_C = 150 \text{ GHz} \)
  - too high to detect directly

- **Axial frequency** \( \nu_Z = 200 \text{ MHz} \)
  - relatively easy to detect

Couple the axial frequency \( \nu_Z \) to the cyclotron energy.

Small measurable shift in \( \nu_Z \) indicates a change in cyclotron energy.

\[
B_z = B_0 + B_2 z^2
\]

\[B \]

\[\text{nickel rings}\]
Couple **Axial Motion and Cyclotron Motion**

Add a “magnetic bottle” to uniform $B$

$$\Delta \vec{B} = B_2[(z^2 - \rho^2 / 2) \hat{z} - z \hat{\rho}]$$

$$H = \frac{1}{2} m\omega_z^2 z^2 - \mu B_2 z^2$$

Spin flip is also a change in $\mu$.
Quantum Non-demolition Measurement

\[ H = H_{\text{cyclotron}} + H_{\text{axial}} + H_{\text{coupling}} \]

\[ [ H_{\text{cyclotron}}, H_{\text{coupling}} ] = 0 \]

**QND:** Subsequent time evolution of cyclotron motion is not altered by additional QND measurements.
An Electron in a Penning Trap

- very small accelerator
- designer atom

Electrostatic quadrupole potential

\[ V = z^2 - \frac{1}{2} (x^2 - y^2) \]

Magnetic field

153 GHz

cool 12 kHz

200 MHz
detect

need to measure for g/2
Cylindrical Penning Trap

Electrostatic quadrupole potential → good enough near trap center
Detection of single electron axial motion

• The axial oscillator is coupled to a tuned-circuit amplifier

• Axial motion is driven to increase signal
Better Detection Amplifier
Detecting a High Axial Frequency

60 MHz → 200 MHz
First One-Particle Self-Excited Oscillator

Feedback eliminates damping

Oscillation amplitude must be kept fixed
  Method 1: comparator
  Method 2: DSP (digital signal processor)

"Single-Particle Self-excited Oscillator"
B. D'Urso, R. Van Handel, B. Odom and G. Gabrielse
Use Digital Signal Processor $\rightarrow$ DSP

- Real time Fourier transforms
- Use to adjust gain so oscillation stays the same
Cyclotron Quantum Jumps to Observe Axial Self-Excited Oscillator

- Use positive feedback to eliminate the damping
- Use comparator to make constant drive amplitude

Measure very large axial oscillations

(again using cyclotron quantum jump spectroscopy)
Observe Tiny Shifts of the Frequency of the Self-Excited Oscillator

Tiny, but unmistakable, changes in the axial frequency signal one quantum changes in cyclotron excitation and spin. How?
Quantum Jump Spectroscopy

- one electron in a Penning trap
- lowest cyclotron and spin states

\[ \begin{align*}
    n = 2 & \quad \overline{f}_c = \overline{v}_c - \frac{3\delta}{2} \\
    n = 1 & \quad \overline{v}_a = \frac{g\nu_c}{2} - \overline{v}_c \\
    n = 0 & \quad m_S = -\frac{1}{2}, \quad m_S = \frac{1}{2}
\end{align*} \]
Gabrielse
One-Electron in a Microwave Cavity

- One-electron
  - cyclotron oscillator
  - within a cavity
  - QND measurement of the cyclotron energy

---

B

n=0
n=1
n=2
n=3

cyclotron energy

time
Cool to Eliminate Blackbody Photons

• one electron
• Fock states of a cyclotron oscillators
• due to blackbody photons

On a short time scale
→ in one Fock state or another
Averaged over hours
→ in a thermal state
Control Inhibition of Spontaneous Emission within a Cavity

• In free space, cyclotron lifetime = 0.1 s
• In our cylindrical trap → 16 s lifetime

How long to make a quantum jump down

![Graph showing decay time vs. number of n=1 to n=0 decays.]

\[ \tau = 16 \text{ s} \]

![Diagram showing axial frequency shift vs. time.]

0 100 200 300

-3 0 3 6 9 12 15

0 10 20 30
Spontaneous Emission is Inhibited

Free Space

\[ B = 5.3 \, \text{T} \]

Within Trap Cavity

\[ B = 5.3 \, \text{T} \]

\[ \gamma = \frac{1}{75 \, \text{ms}} \]

\[ \gamma = \frac{1}{16 \, \text{sec}} \]

Inhibited By 210!
Some Challenges
Big Challenge: Magnetic Field Stability

- Large magnetic field (5 Tesla) from solenoid
- Nuclear magnetism of trap apparatus

Experimenter’s “definition” of the g value:

\[
\frac{g}{2} = \frac{\omega_s}{\omega_c} = 1 + \frac{\omega_a}{\omega_c}
\]

Relatively insensitive to B

But: problem when B drifts during the measurement

- Large magnetic field (5 Tesla) from solenoid
- Nuclear magnetism of trap apparatus
Magnetic Field – Takes Months to Settle after a Change

Two months is a long time to wait for the magnetic field to settle.
Self-Shielding Solenoid Helps a Lot

Flux conservation $\Rightarrow$ Field conservation
Reduces field fluctuations by about a factor $> 150$

“Self-shielding Superconducting Solenoid Systems”,
A tabletop experiment ...  
if you have a high ceiling
Eliminate Nuclear Paramagnetism

One Year Setback

Deadly nuclear magnetism of copper and other “friendly” materials

→ Had to build new trap out of silver
→ New vacuum enclosure out of titanium ~ 1 year
→ …
Silver trap improvement

- The new silver trap decreases T-dependence of the field by ~ 400
- With the silver trap, sub-ppb field stability is “easily” achieved
New Silver Trap
“In the Dark” Excitation $\rightarrow$ Narrower Lines

1. Turn FET amplifier off
2. Apply a microwave drive pulse of $\sim$150 GHz (i.e. measure “in the dark”)
3. Turn FET amplifier on and check for axial frequency shift
4. Plot a histograms of excitations vs. frequency

- Good amp heat sinking, amp off during excitation $T_z = 0.32$ K
Quantum Jump Spectroscopy

Lower temperature so there are no blackbody photons in the cavity

→ Introduce microwave photons into the trap cavity – near resonance with the cyclotron frequency

Look for resonance

Count the quantum jumps/time for a particular drive frequency
Measurement Cycle

\[ g = \frac{\omega_s}{2 \omega_c} = 1 + \frac{\omega_a}{\omega_c} \]

simplified

3 hours
1. Prepare \( n=0, m=1/2 \) \( \rightarrow \) measure anomaly transition
2. Prepare \( n=0, m=1/2 \) \( \rightarrow \) measure cyclotron transition

0.75 hour
3. Measure relative magnetic field

Repeat during magnetically quiet times
**Measured Line Shapes for g-value Measurement**

**It all comes together:**
- Low temperature, and high frequency make narrow line shapes
- A highly stable field allows us to map these lines

**Precision:**
Sub-ppb line splitting (i.e. sub-ppb precision of a g-2 measurement) is now “easy” after years of work
Cavity Shifts

Uncertainty completely limited now by how well we understand the cavity shifts
Cavity Shifts of the Cyclotron Frequency

\[ \frac{g}{2} = \frac{\omega_s}{\omega_c} = 1 - \frac{\omega_a}{\omega_c} \]

Within a Trap Cavity

\[ \gamma = \frac{1}{16 \text{ sec}} \]

spontaneous emission inhibited by 210

\[ B = 5.3 \text{ T} \]

cyclotron frequency is shifted by interaction with cavity modes
Cylindrical Penning Trap

Good approximation to a cylindrical microwave cavity
- Good knowledge of the field within
- Challenge is to make a sufficiently good electrostatic quadrupole

Holes and slits to make a trap
- Must still measure resonant EM modes of the cavity

have done this
Cavity modes and Magnetic Moment Error

Operating between modes of cylindrical trap where shift from two cavity modes cancels approximately

first measured cavity shift of g
### Summary of Uncertainties for $g$ (in ppt = $10^{-12}$)

Test of cavity shift understanding

Measurement of $g$-value

<table>
<thead>
<tr>
<th>Source</th>
<th>$\bar{\nu}_c$</th>
<th>146.8 GHz</th>
<th>149.0 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\nu}_z$ shift</td>
<td>0.2 (0.3)</td>
<td>0.00 (0.02)</td>
<td></td>
</tr>
<tr>
<td>Anomaly power</td>
<td>0.0 (0.4)</td>
<td>0.00 (0.14)</td>
<td></td>
</tr>
<tr>
<td>Cyclotron power</td>
<td>0.0 (0.3)</td>
<td>0.00 (0.12)</td>
<td></td>
</tr>
<tr>
<td>Cavity shift</td>
<td>12.8 (5.1)</td>
<td>0.06 (0.39)</td>
<td></td>
</tr>
<tr>
<td>Lineshape model</td>
<td>0.0 (0.6)</td>
<td>0.00 (0.60)</td>
<td></td>
</tr>
<tr>
<td>Statistics</td>
<td>0.0 (0.2)</td>
<td>0.00 (0.17)</td>
<td></td>
</tr>
<tr>
<td>Total (in ppt)</td>
<td>13.0 (5.2)</td>
<td>0.06 (0.76)</td>
<td></td>
</tr>
</tbody>
</table>
Spinoff Measurements Planned

- Use self-excited antiproton oscillator to measure the antiproton magnetic moment → million-fold improvement?

- Compare positron and electron g-values to make best test of CPT for leptons

- Measure the proton-to-electron mass ratio directly
Emboldened by the Great Signal-to-Noise

Make a one proton (antiproton) self-excited oscillator
   ➔ try to detect a proton (and antiproton) spin flip

   • Hard: nuclear magneton is 500 times smaller
   • Experiment underway ➔ Harvard
      ➔ also Mainz and GSI (without SEO)
         (build upon bound electron g values)

   ➔ measure proton spin frequency
   ➔ we already accurately measure antiproton cyclotron frequencies
   ➔ get antiproton g value (improve by factor of a million or more)
Summary and Conclusion
How Does One Measure $g$ to 7.6 Parts in $10^{13}$?

**New Methods**
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\( 7.6 \times 10^{-13} \)

- First improved measurement since 1987
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\[ \alpha^{-1} = 137.035\,999\,710 \pm 0.000\,000\,096 \quad 7.0 \times 10^{-10} \]

- First lower uncertainty since 1987
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Stay Tuned.

We Have Some More Ideas for Doing Better