A new determination of the fine structure constant based on Bloch oscillations of 87 Rb atoms in a vertical optical lattice

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Determinations of the fine structure constant $\alpha$

- Codata98
- Codata02

$$\Gamma' \quad p, h-90 \quad \text{quantum Hall effect}$$

- hfs muonium

- $R_K = h/e^2 = \mu_0 c / 2\alpha$

- $\alpha = \frac{e^2}{4\pi \varepsilon_0 \hbar c}$

- $\alpha^2 = \frac{2 R_\infty A_r(X)}{c A_r(e) m(X)} \hbar$

• Two photon transition: defined momentum transfer
• Low spontaneous emission
• Doppler sensitive
• Controllable width of the transition (pulse duration):

\[ \Omega_{\text{eff}} \propto \frac{\Omega_1 \Omega_2}{\Delta} \]

Principle of our experiment

- selection of an initial sub-recoil velocity class
- coherent acceleration: $N$ Bloch oscillations, momentum transfer $2N\hbar k$
- measurement of the final velocity class

Final uncertainty: $\sigma_{vr} = \sigma_v / (2N)$
1) Subrecoil velocity selection

Determination of the center of velocity distribution

3) Velocity measurement

⇒

F=1

F=2

⇒

\( \sim \delta_{\text{center}} \)
$^8\text{Rb}$

5$P_{3/2}$

5$S_{1/2}$

MOT + Optical molasses

\[ \delta_{\text{sel}} \]

\[ \pi\text{-pulse} \]

\[ \delta_{\text{meas}} \]

\[ \pi\text{-pulse} \]

Blow away beam

Tuned Frequency

Detection

Velocity sensor

\[ F=1 \rightarrow F=2 \]

\[ F=2 \rightarrow F=1 \]
Velocity measurement: results

- intensity: 100 mW.cm$^{-2}$; detuning $\Delta \sim 1050$ GHz
- 1.8 s per point (loading of the MOT)
- 300 points (~10 minutes)

$\sigma_v \approx 1$ Hz $\rightarrow \frac{v_r}{15000}$
Reduction of constant systematic shifts

- Light shift
- Quadratic Zeeman shift

\[ \Delta v \]

\[ \Delta \text{Zeeman} \]
Compensation of energy shifts by inverting the direction of Raman beams

Two spectra → one velocity measurement

Reduction of systematic shifts

- Light shift
- Quadratic Zeeman shift
Coherent acceleration of atoms

Raman transitions in a same internal state

Laboratory frame

\[ 2\pi \delta' = 2\pi (\nu_1 - \nu_2) = 2k a t \]

\[ \nu_1 \nu_2 \]

coherent acceleration, 2 ~k per cycle

Acceleration

\[ v_B = \frac{mg}{2\hbar k} \]

Stationary
Coherent acceleration of atoms

Dressed atom

Bloch oscillations in the fundamental energy band

Accelerated frame

Stationary experiment

- Differential measurement to cancel systematic effects
- Atomic cloud don’t move between selection and measurement
  → g measurement at the same place

\[
v_i = g(t_i - N_i \tau_B)
\]

\[
g = \frac{(v_1 - v_2)}{v_r} + 2(N_1 - N_2)
\]

\[
\tau_B = \frac{2\hbar k}{mg}
\]

\[
\tau_B = 1.2 \text{ ms and } U_0 = 2.7 \text{ E}_r
\]

Measurement of \(g/v_r\) à 10^{-6}

**Experimental sequence (acceleration)**

**Selection:**
F=2 → F=1

**Measurement:**
F=1 → F=2

Velocity variation

\[ \Delta V^\pm = gt \pm 2Nv_r \]

- \( t \): time between the two Raman pulses
- \( N \): Number of Bloch oscillations

Acceleration in both opposite directions

\[ v_r = \frac{\Delta V^+ - \Delta V^-}{4N} \]
Experimental set up

- Cold atomic cloud
- Detection beams
- Signal from $F=2$
- Signal from $F=1$
- Repumping beam
- Partially reflecting plate
- $\lambda/2$ wave plate
- Blow-away beam
- Upper optical fiber
- Bloch beam
- Lower optical fiber
- Raman beam 1
- Raman beam 2
- Blow-away beam
- Bloch beam
- Partially reflecting plate
Results

Transfer efficiency: > 99.95% per oscillation (2 recoils)

About 450 Bloch oscillations in each direction: 1800 recoils

Measurements performed in April 2005

1 point = 4 spectra

Statistical uncertainty on $\alpha = 4.4 \times 10^{-9}$

$\alpha^{-1} = 137.035\,998\,78\ (91)$

Uncertainty $6.7 \times 10^{-9}$

Systematic effects

- **Lasers frequencies**: FP cavity → uncertainty 300kHz → $u_r(\alpha) = 8 \times 10^{-10}$

- **Beams misalignment**: Optical fibers to couple Raman/Bloch beams into the cell

  - maximum misalignment:
    \[
    \theta_r = 3 \times 10^{-5} \text{ rad} \\
    \theta_B = 1.6 \times 10^{-4} \text{ rad}
    \]

  - Correction on $\alpha^{-1}$: $-(2 \pm 2) \times 10^{-9}$
Systematic effects

Gravity gradient:
- R: Earth radius
- t: spacing time / sel-meas = 12 ms

\[ \text{Correction on } \alpha^{-1} \sim - \frac{g t^2}{R} \sim 10^{-10} \]

Level shifts:
- Light shift
  - Expansion of the cloud
    \[ \Delta I = 10\% \text{ when } k^2_R \leftrightarrow k^1_R \]
  - Magnetic field gradient = trajectory effect
    \[ \Delta z = 0.3 \text{mm when } k^2_R \leftrightarrow k^1_R \]

\[ \text{Correction on } \alpha^{-1} \sim (6.6 \pm 2) \times 10^{-9} \]

Quadratic magnetic force:

\[ \text{Correction on } \alpha^{-1} \sim \frac{(F/M)t}{2 N v_r} \sim (-1.3 \pm 0.4) \times 10^{-9} \]
What is the momentum transferred to the atoms by laser beams?

Gaussian beam : Plane waves superposition : \( k_{\parallel}^2 = \frac{\omega^2}{c^2} - k_{\perp}^2 < \frac{\omega^2}{c^2} \)

Momentum transferred = gradient of the phase

\[ E(r, z) = E(r)e^{i\phi(r,z)} \quad p \rightarrow p + \hbar k_{\text{eff}} \quad \text{avec} \quad k_{\text{eff}}(r, z) = \partial_z \phi(r, z) \]

Gaussian beam:

\[ \phi(r, z) = k z - \phi_G(z) + k \frac{r^2}{2R} \]

Gouy phase

Curvature radius

\[ k_{\text{eff}} = k - \frac{2}{k w^2(z)} \frac{r^2}{2R^2} \frac{dR}{dz} \times k \]

\( k_{\text{eff}} \) can be measured with a wave front analyzer \((R, w)\)
Recoil transmitted by one Bloch oscillation: $2\sim k$ or $2n\sim k$?
Doppler effect for the Raman transitions: $2kv$ or $2nkv$?

$$\frac{n-1}{\pi \rho} = \frac{\Gamma}{\Delta} \left( \frac{\lambda}{2\pi} \right)^3$$

- $\rho$: density
- $\Gamma$: natural width
- $\Delta$: detuning

For the cold atoms
Initial atomic density: $10^{11}$ atoms/cm$^3$
Raman beams: $\Delta = 1050$ GHz:
  - $(n-1) = 4.10^{-10}$ (selection)
  - $(n-1) < 10^{-12}$ (measure)

Bloch beams: $\Delta = 40$ GHz:
  - $(n-1) = 2.10^{-10}$ (selected atoms)

For the background vapor
density: $8.10^8$ atoms/cm$^3$  $(n-1) \sim 4.10^{-10}$
**Index of refraction**

*PRL 94 170403 (2005) (MIT): Photon Recoil Momentum in Dispersive Media*

Observation: modification of recoil energy in a dispersive medium (BEC).

\[ N_1 \ll N_{\text{tot}} \]

\[ 2n \sim k \]

Dispersive medium \quad Atoms

\[ N_{\text{tot}} \quad N_0 \quad N_1 \]

**n**: index of refraction

\[ N_1 \ll N_{\text{tot}} \]

\[ 2(1-n)\frac{N_1}{N_0} \sim k \]

\[ 2n \sim k \]

---

**Bloch oscillations**:

\[ p_{\text{final}} = 2n \sim k + 2(1-n) \sim k \frac{N_{\text{tot}}}{N_{\text{tot}}} = 2 \sim k \quad \text{if } \eta = 100\% \]

Accelerated atoms \(\Leftrightarrow\) dispersive medium

otherwise \(\sim (1-\eta)(n-1)\)

---

**Raman transition**:

Atomic cloud

\[ \omega' = \omega - nk v_{\text{atom}} + (n-1)k v_{\text{medium}} \]

\[ \omega' = \omega - k v_{\text{atom}} + (n-1)k(v_{\text{medium}} - v_{\text{atom}}) \]

\[ \frac{dL}{dt} = 0 \Leftrightarrow v_{\text{medium}} = v_{\text{atom}} \quad \text{no effect} \]
### Error budget

<table>
<thead>
<tr>
<th>Source</th>
<th>Correction $(\alpha^{-1})(\text{ppb})$</th>
<th>Uncertainty $(\alpha^{-1})(\text{ppb})$</th>
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<tbody>
<tr>
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<td>✓ Beams alignment</td>
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<td>✓ Wave front curvature and Gouy phase</td>
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<td>✓ Index of refraction (background vapor)</td>
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<td><strong>TOTAL</strong></td>
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</tbody>
</table>

Cladé et al. (paper in preparation) $\alpha^{-1} = 137.035\,998\,84 \,(91)$
Improvements in progress in Paris

Goal: $10^{-9}$ uncertainty

- anti vibration platform
- magnetic shielding
- new cell and slow atom source
- ...

2D MOT

MOT + molasse

μ-metal boxes