Breaking The Unbreakable: 
Violation of Lorentz invariance, CPT and variation of couplings in time

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Plan

1. Introduction and motivations. Lorentz violation and change of $\alpha$ as probes of new infrared physics.

2. Effective theories of Lorentz violation. Kostelecky Lagrangian and higher-dimensional operators. ”Lorentz violation” from scalar-tensor theories of gravity.

3. CPT-odd EDMs.

4. Bekenstein-like models of ”changing alpha”.

Three ways of probing new physics

1. High-energy colliders

Probing

\[ \ell \sim \frac{1}{q} \sim \frac{1}{E} \]

To probe/discover new physics at scale \( \Lambda_{NP} \) a typical energy of \( E \sim \Lambda_{NP} \) is required.

2. Precision measurements at low energies

\[ \Delta \text{Energy} \sim \frac{m^{n+1}}{\Lambda_{NP}^n}, \]

where typically \( n > 0 \).

3. Cosmological/astrophysical probes: CMB anisotropy, BBN, dark matter, star cooling rates, late decays of massive particles, etc.

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Different types of precision measurements

Probing new UV physics

1. $g - 2$ of the muon, electron
2. $CP$ asymmetries and rare decays of $K, B$
3. Parity nonconservation in atoms
4. Proton decay
5. Neutrino oscillations
6. Lepton flavor violation
7. EDMs
   ......

Probing new IR physics

1. Neutrino oscillations
2. Axion searches
3. Searches of preferred frame (”Lorentz violation”)
4. Searches of $\alpha(t), m(t)$
5. Searches of new gravitational interactions (”fifth force”)
6. Tests of equation of state for ”dark energy”
Change of $\alpha = \text{new infrared physics}$

If a coupling depends on a space-time coordinate, it will also have a kinetic term, $(\partial \text{coupling}/\partial x)^2$, and possibly self-potential $V(\text{coupling})$. If ”coupling” changes over macroscopic time $T$, it means that it is a light (IR) scalar degree of freedom.

$$
\frac{1}{g^2} F_{\mu\nu} F^{\mu\nu} \rightarrow \frac{1}{g^2(t)} F_{\mu\nu} F^{\mu\nu} \rightarrow \\
\frac{1}{g^2(x)} F_{\mu\nu} F^{\mu\nu} \rightarrow \frac{1}{g_0^2} B_F(\phi(x)) F_{\mu\nu} F^{\mu\nu}
$$

Detection of non-zero $\Delta \alpha(t)$ would naturally imply the presence of new IR degrees of freedom other than SM.

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"Lorentz violation" = new infrared physics

The possibility of space-time anisotropy often called "Lorentz violation" can be parametrized by constant vectors, tensors, etc. (Kostelecky,...). The statement $b_\mu = \text{const}$ does not make sense in general relativity. $b_\mu$ is a dynamical field, $b_\mu(x)$, with Goldstone modes that correspond to change of $b_0, b_i$ at constant $b_\mu b^\mu$.

Detection of non-zero $b_\mu, \theta_{\mu\nu},...$ would naturally imply the presence of new IR degrees of freedom other than SM.

If the Compton "wavelength" of new IR degree of freedom is comparable to the size of the observed Universe, then this IR degree of freedom contributes to dark energy.
Cosmological motivations. Dark energy.

\(~75\%\) of total energy density is in the form of dark energy, that does not cluster, almost does not depend on time, and has pressure \(\approx -\)energy.

Two candidates for dark energy
1. Cosmological constant - no new effects other than cosmology.
2. Quintessence (IR scalar field) - could couple to matter and be detected by other means.

The most crucial test is \(w = -1\) relation which holds exactly for cosmological constant, and is expected to be violated at some level for quintessence.
Presently, \(w\) is consistent with \(-1\) at 10\% level, and \(dw/dt\) is consistent with 0.
IR scalar-induced effects

\[ \mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) + \bar{\psi}(\partial_{\mu} \gamma^{\mu} - m_e) \psi - \phi \frac{m_e}{M_s} \bar{\psi} \psi - \phi \frac{m_e}{M_p} \bar{\psi} i \gamma_5 \psi \]

If \( \phi \) is light, i.e. quintessence-like field, then there is a preferred frame where \( \partial_{\mu} \phi = (\partial_t \phi, 0, 0, 0) \), that quite generically coincides with the frame of CMB. \( \partial_t \phi \) is limited by \((\rho_{d,e}(1+w))^{1/2}\). There are several consequences of the \( \phi - \psi \) interaction Lagrangian:

1. Particle mass depends on time: \( m_{eff}(t) = m_e(1 + \phi/M_s) \)
2. There is an additional Zeeman-like splitting from \( H_{int} = M_p^{-1} \vec{S} \cdot \nabla \phi \). If the spin moves with velocity \( v \) over the CMB frame, then \( \nabla \phi = \vec{v} \dot{\phi} \).
3. Surrounding mass will create additional \( \nabla \phi \) which is roughly parallel to local acceleration. It contributes to gravitational acceleration on top of the graviton contribution, may violate the universality of gravitational force and creates the Zeeman splitting in the direction of local gravitational acceleration \( \vec{S} \cdot \vec{g} \).

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Searches of Lorentz and CPT violation

Laboratory Experiments

1. Clock comparison experiments (Drever-Hughes type).
   Search of the spatial anisotropy.

2. Michelson-Morley type of experiments.

3. CPT tests with neutral Kaons; with $e^+e^-$; with antihydrogen

....

Astrophysical and Cosmological searches

1. LV signatures in gamma-ray bursts

2. LV signatures in cosmic rays

3. Rotation of polarisation plane over cosmological distances

4. Change of couplings in time/space
1. **String theory** can have Lorentz-non-trivial backgrounds. Non-zero $B_{\mu\nu}$ field $\rightarrow$ non-commutative field theory.

2. “Quantum gravity” speculations about modified dispersion relations

$$E^2 = p^2 + m^2 + c_1 \frac{E^3}{M_{\text{Pl}}} + c_2 \left( \frac{E^4}{M_{\text{Pl}}^2} \right) + \ldots$$
Kostelecky et al., Coleman and Glashow parametrization of anisotropy

Dimension 3 operators:

\[ \mathcal{L}^{(3)}_{\text{QED}} = -a_{\mu} \bar{\psi} \gamma_{\mu} \psi - b_{\mu} \bar{\psi} \gamma^{\mu} \gamma_{5} \psi - \frac{1}{2} H_{\mu\nu} \bar{\psi} \sigma^{\mu\nu} \psi - k_{\mu} \epsilon^{\mu\nu\alpha\beta} A_{\nu} \frac{\partial}{\partial x^{\alpha}} A_{\beta}, \]

Dimension 4 operators:

\[ \mathcal{L}^{(4)}_{\text{QED}} = -c_{\mu\nu} \bar{\psi} \gamma_{\mu} \partial_{\nu} \psi - d_{\mu\nu} \bar{\psi} \gamma_{\mu} \gamma_{5} \partial_{\nu} \psi - k_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}, \]

Dimension 5 operators, Myers, MP; Bolokhov, MP

Dimension 3 operators give spin precession signatures. Dimension 4 operators give different speeds of propagation for electrons and gamma, and can be searched for in astrophysics.

If \( a, b, c, d, k \) are fundamental parameters coming from some UV scale \( \Lambda \), why would not all dimension three coefficients be \( O(\Lambda) \) and dimension 4 be \( O(1) \)?
Dimension five operators

Myers, Pospelov

Dimension 5 operators are severely constrained by these selection rules. The only possible operators that give $E^3$ modification of the dispersion relations are

Scalars:
\[ \mathcal{L}_s = i \frac{\kappa}{M_{Pl}} \bar{\Phi} (n \cdot \partial)^3 \Phi \]

Vectors:
\[ \mathcal{L}_\gamma = \frac{\xi}{M_{Pl}} n^a F_{ad} n \cdot \partial (n_b \tilde{F}^{bd}) \]

Fermions:
\[ \mathcal{L}_f = \frac{1}{M_{Pl}} \bar{\Psi} \left( \eta_1 \eta + \eta_2 \gamma_5 \right) (n \cdot \partial)^2 \Psi \]

There are no modifications to dispersion relation for a real scalar!
For photons, this modification is helicity dependent,
\[ \left( E^2 - p^2 \pm \frac{2\xi}{M_{Pl}} p^3 \right) (\epsilon_x \pm i\epsilon_y) = 0 \]

Only three constants are relevant for QED, $\eta_{1,2}$ and $\xi$. 

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Phenomenological constraints

Preferred frame → loss of isotropy due to Earth’s motion relative to a “fixed” frame, \( n_i \simeq v_i/c \sim 10^{-3} \).

A typical spin precession constraint is then

\[
|\eta_d - \eta_Q - 0.5(\eta_u - \eta_Q) + 10^{-3}\xi| \leq 10^{-8}
\]

(See also, Sudarsky, Urrutia, Vucetich, Phys. Rev. Lett.89:231301,2002)

The same operators are used to derive constraints from astrophysics (Jacobson, Liberati, Mattingly), and from the precession of polarization of light over cosmological distances (Gleiser, Kozameh). The limits are typically at \( O(10^{-2} - 10^{-5}) \) level.

Much stronger constraints from the mere existence of high-energy cosmic rays have been reported recently in Gagnon, Moore, hep-ph/0404196.

Astrophysics and low-energy tests provide constraints on different linear combinations of parameters with sensitivity better than \( 1/M_{Pl} \) level. All astrophysics constraints arise because the dispersion relation is modified.
Mocioiu, Pospelov, Roiban

\[[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}\]. One can rewrite theory in normal coordinates in terms of the \(*\)-product,

\[(\phi \ast \psi)(x) = \exp\{\frac{i}{2}\theta^{\mu\nu}\partial_\mu(x)\partial_\nu(y)\}\phi(x)\psi(y)|_{x=y}\]

\(\theta^{\mu\nu}\) has the dimension of inverse energy squared, that I call \(1/\Lambda_{NC}^2\).

In the linear order in \(\theta\), there are corrections in terms of dimension 6 operators,

\[\mathcal{L}(\ast) = \mathcal{L} + \theta^{\mu\nu}\sum_{\mu\nu} O_{\mu\nu}\]

For example, in NC QED,

\[\Delta \mathcal{L} = \frac{e}{8}\theta^{\mu\nu} F_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} - \frac{e}{2}\theta^{\mu\nu} F_{\mu\alpha} F^{\alpha\beta} F_{\beta\nu} + \ldots\]

The “magnetic” part of \(\theta^{\mu\nu}, \theta_{ij} \equiv \epsilon_{ijk}(\theta_B)_k\), couples to \((\theta_B \cdot B)(E^2 - B^2)\). In a bound state of charged constituents with a total spin \(S\) (nucleon, nuclei) this would lead to an effective coupling \(\theta_B \cdot S\),

\[\langle N | \mathcal{L}_{QCD}(\ast) | N \rangle \simeq d_\theta \theta^{\mu\nu} \bar{N}\sigma_{\mu\nu} N,\]

where \(d_\theta \simeq 0.1(\text{GeV})^3\). Nucleon energy depends on the orientation of its spin relative to ther preferred frame given by \(\theta^{\mu\nu}\).
Clock comparison experiments

As the Earth rotates, the effective angle between laboratory $B$ and $\theta_B$ changes, leading to $24\text{hr}$ modulation of the precession frequency.

![Diagram of Earth and Sun with vectors b_i, V_CMB, and b_i](image)

Best sensitivity measurements compare different spins in the magnetic field. C. Berglund et al., (1995) uses $^{133}\text{Cs}$ and $^{199}\text{Hg}$. $\theta_B$ affects mostly the nuclear spin, so that

$$\frac{\Delta \omega_{\text{Hg}}}{\omega_{\text{Hg}}} \gg \frac{\Delta \omega_{\text{Cs}}}{\omega_{\text{Cs}}}$$

The LV spin precession is not found with accuracy $100 \text{ nHz} \approx 10^{-31}\text{GeV}$, so that

$$\frac{0.1(\text{GeV})^3}{\Lambda_{NC}^2} < 10^{-31}\text{GeV} \rightarrow \Lambda_{NC} > 10^{15} \text{ GeV}$$

An improvement by up to $10^4$ is claimed to be possible with the new $K^{-}\text{He}$ co-magnetometer at Princeton (M. Romalis). Best limits on LV effects are obtained in D. Bear et al., (2000)
Naturalness problem

We assumed dimension 5 operators without checking whether dimension 3 exist. They do! See e.g. papers by A. Kostelecky. Again, for QED,

\[ \mathcal{L}_{\text{QED}}^{(3)} = -b_\mu \bar{\psi} \gamma^\mu \gamma_5 \psi - \frac{1}{2} H_{\mu\nu} \bar{\psi} \sigma^{\mu\nu} \psi - k_\mu \epsilon^{\mu\nu\alpha\beta} A_\nu \frac{\partial}{\partial x^\alpha} A_\beta, \]

Dimension three LV operators can be induced from dimension 5 via quantum loops with quadratically divergent coefficients

\[ b_\mu \sim (\text{loop factor}) \times \xi \frac{\Lambda_{UV}^2}{M_{\text{Pl}}}. \]

It is a disaster unless either fine-tuning happens, or \( \Lambda_{UV}^2 \)-divergence is absent, or the cutoff scale is moderate to low.

Supersymmetry saves the day as it forbids LV at dimension 3 and 5 level (Groot Nibbelink, MP). After the (soft) breaking of SUSY, \( \text{dim } 3 = \text{loop} \times m_{\text{soft}}^2 \times \text{dim } 5 \).

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Benchmarks for Lorentz Violation?

- In Kostelecky approach, the benchmark values for dim3 operators at the level of $m_{\text{weak}}^2/M_{\text{Pl}}$, $m_p^2/M_{\text{Pl}}$ inspired by dimension 5 operators $\sim 1/M_{\text{Pl}}$ have been long passed with nil result.

- In models with quintessence field driven by self-potential and pseudoscalar interactions with particles normalized on Planck scale, the benchmark value for effective $\vec{b}$ is $vH \sim 10^{-45}$ GeV (or $10^{-21}$Hz). Out of reach.

- In models with scalar-tensor gravity (Brans-Dicke-like) plus pseudoscalar interaction the size of Zeeman splitting relative to vertical direction is

$$h\nu = \frac{M_s}{M_p} \frac{g}{2\pi \omega_{BD}^{1/2}} \approx \frac{M_s}{M_p \omega_{BD}^{1/2}} \times 5 \text{ nHz}$$

5nHz is a benchmark value for coupling of a spin to $\vec{g}$ and it can and should be probed.
CPT-odd EDMs

Bolokhov, MP, Romalis. EDMs test P and T, and thus can test CPT-odd but CP-even interactions

\[ H_{T,P-odd} = -dE \cdot \frac{S}{S} \rightarrow \begin{cases} \mathcal{L} = -d_2 \bar{\psi} \sigma^{\mu\nu} \gamma_5 \psi F_{\mu\nu}, & \text{CP}, \text{CPT}+ \\ \mathcal{L} = \bar{\psi} \gamma^\mu \gamma_5 \psi F_{\mu\nu} n^\nu, & \text{CP}, \text{CPT} - \end{cases} \]

where \( n^\mu = (1, 0, 0, 0) \) is the “preferred frame”. \( n^\mu \neq 0 \) violates Lorentz invariance and enables violation of CPT.

For a generic spin

\[ \mathcal{L}_{\text{EDM}} = F_{\mu\nu} s^\nu (d_{\text{CP}} u^\mu + d_{\text{CPT}} n^\mu). \]

For a quark

\[ \bar{\psi}_q (e Q_i F_{\mu\nu} + g_s t^a G_{\mu\nu}^a) \gamma^\nu \gamma_5 \psi_q = -i \bar{\psi}_q [D_\mu, D_\nu \gamma^\nu \gamma_5] \psi_q = 2m_i \bar{\psi}_q D_\mu \gamma_5 \psi_q = m_q \bar{\psi}_q [D_\nu \gamma^\nu, \gamma_5] \psi_q = 0. \]

For an electron there is an exact Schiff theorem,

\[ \bar{\psi}_e F_{\mu\nu} \gamma^\nu \gamma_5 \psi_e = 0. \]

For electrons CPT-odd EDM cancels out, while for a quark it does not:

\[ \mathcal{L}_{\text{CPT}} = \sum_{i=u,d,s} d_i^\mu \bar{q}_i \gamma^\lambda \gamma^5 F_{\lambda\mu} q_i. \]

A much simpler \( \mathcal{L}_{\text{eff}} \).
Distinction between CP and CPT odd EDMs

- Patterns of $CPT$-odd EDMs:
  1. non-zero $d_n$, given by QCD matrix elements of the axial-vector charges of quarks: $d_n = 0.8d_d^0 - 0.4d_u^0 - 0.1d_s^0$.
  2. Zero $d_e$ and $d_{muon}$, severe suppression of EDMs of paramagnetic atoms.
  3. EDMs of diamagnetic atoms are induced by the EDMs of (valence) nucleons.
  4. Suppressed signal from deuteron EDM in the storage ring relative to $CP$-odd case.

- Linear decoupling of $CPT$-odd EDM with the heavy scale. Current limits probe $10^{12}$ GeV. $d_n$ can probe $CPT$ violation as small as $1/M_{\text{Planck}}$.

- Some Lorentz-noninvariant signatures of $d_{CPT}^\mu$ are expected, including a modulation of the EDM signal.

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Experimental hint on changing couplings?

Webb et al., $z = 1 - 3$

$$\Delta(\alpha)/\alpha = (-1.9 \pm 0.5) \times 10^{-5}. \quad (1998)$$

$$\Delta(\alpha)/\alpha = (-0.72 \pm 0.18) \times 10^{-5}. \quad (2001)$$

$$\Delta(\alpha)/\alpha = (-0.543 \pm 0.116) \times 10^{-5}. \quad (2003)$$

See, however, Chand et al., 2004, $z = 0.5 - 2.5$

$$\Delta\alpha/\alpha = (-0.06 \pm 0.06) \times 10^{-5}.$$

See also this year claim:

$$\Delta(m_e/m_p)/(m_e/m_p) = (2.0 \pm 0.6) \times 10^{-5}.$$

In models of interacting IR scalar field, the change of couplings/masses is a more sensitive probe than anisotropic spin interactions. However, there is a tight constraint from the fifth force.
Minimal Bekenstein model of changing $\alpha$

\[
S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} M_{Pl}^2 R + \frac{1}{2} (\partial \mu \phi)^2 - \frac{1}{4} (1 + \frac{\zeta_F}{M_*} \phi) F_{\mu \nu} F^{\mu \nu} \right]
\]

Defining $\phi(z = 0) = 0$, we have

\[
\frac{\alpha_0 - \alpha(t)}{\alpha} = \frac{\zeta_F}{M_*} \phi
\]

The cosmological evolution of $\phi$ is determined by the same coupling.

\[
\zeta_N m_N \bar{N} N = \langle N | \frac{1}{4} F_{\mu \nu} F^{\mu \nu} | N \rangle
\]

$\zeta_p = -0.0007 \zeta_F; \quad \zeta_n = 0.00015 \zeta_F$

Cosmological equation on $\phi$

\[
\ddot{\phi} + 3H \dot{\phi} = -\frac{\zeta_m}{M_*} \rho_m,
\]

where $\zeta_m \rho_m = \zeta_{DM} \rho_{DM} + \rho_b (Y_p \zeta_p + Y_n \zeta_n)$. 

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For the minimal model, the contribution from dark matter is neglected.

\[
\frac{\alpha_0 - \alpha(t)}{\alpha} \simeq 10^{-3} \frac{M_{Pl}^2}{M_*^2} \zeta_F^2 (\ln(1 + z) - 1)
\]

On the other hand, composition-dependence of the gravitational force mediated by \(\phi\) requires that

\[
\frac{M_{Pl}^2}{M_*^2} \zeta_F^2 < 10^{-6} \rightarrow \frac{\Delta \alpha}{\alpha} < 10^{-9}
\]

Minimal Bekenstein model predicts \(\Delta \alpha/\alpha\) three-to-four orders of magnitude smaller than Webb et al. claim (and of the different sign).

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Δα in Bekenstein-like models

Allowing for a much larger coupling to dark matter and/or self-potential \( V(\phi) \) could match the claim of Webb et al. Olive, MP

\[ \frac{\Delta \alpha}{\alpha} \]

\[ \text{Log}_{10}(z) \]

\[ \Delta \alpha / \alpha \]

\[ z=0.5 \quad z=3.5 \]

\( \zeta_F = 10^{-5} \), (a) \( \zeta_m = 1, \zeta_\Lambda = 0 \); (b) \( \zeta_m = 1, \zeta_\Lambda = -2 \); (c) \( \zeta_m = 0, \zeta_\Lambda = 1 \).
Qunitessence coupled to EM

Copeland, Nunes, MP

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + V(\phi) - \frac{1}{4} (1 + \frac{\zeta_F}{M_*} \phi) F_{\mu\nu} F^{\mu\nu} \]

Dynamics of \( \phi \) is set by an arbitrary \( V(\phi) \), which allows to fit any set of data.

\[ z = 1 \] quasar absorption probes, \( z = 0.14 \) Oklo phenomenon probe, and \( z = 0 \) laboratory probes of \( \Delta \alpha/\alpha \) can be completely uncorrelated in a sense that \( d\alpha/dt \neq \text{const} \).

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Conceptual problems for of LV and $\Delta\alpha$

1. Spontaneous (vacuum) breaking of Lorentz Invariance??

In analogy with

$$V_{Higgs} = \lambda (H^2 - v^2)^2$$

why not

$$V_{A\mu} = \lambda (A_{\mu}A^\mu - v^2)^2?$$

A. The energy is not positive-definite.
B. The theory does not have proper ultraviolet completion. Cannot be represented by the low-energy limit of some renormalizable theory.

A partial success with $V_{\partial_{\mu}\phi} = \lambda ((\partial_{\mu}\phi)^2 - c^2)^2$.

2. Interacting quintessence at loop level?

At loop level $V(\phi)$ receives quantum corrections from

$$\frac{\phi}{M_*} \langle F_{\mu\nu}F^{\mu\nu} \rangle_{\text{vac}};$$

$$\left(\frac{\phi}{M_*}\right)^2 \langle d^4x F_{\mu\nu}F^{\mu\nu}(x), F_{\mu\nu}F^{\mu\nu}(0) \rangle_{\text{vac}};$$

... 

A. These integrals UV diverge $\rightarrow$ perturbative calculation does not make sense.
B. Being cutoff at any UV particle physics scale $\Lambda$, the resulting corrections to $V(\phi)$ are much stiffer than $V(\phi)$ itself.
C. Unlike the cosmological constant problem where the fine tun-
ing of one number is required, a whole series of fine-tunings with $\phi$, $\phi^2$, $\phi^3$ etc. terms is needed.

Should we worry about this or hope that whatever fixes $\Lambda$ would also fix $V(\phi)$?
Conclusions

1. Searches of space-time anisotropy ("Lorentz violation"), changes of masses and couplings in time/space, fifth force experiments test for the presence of new IR degrees of freedom beyond SM and spin-2 gravity. Such searches are motivated by the known existence of dark energy.

2. One can use effective field theory to parametrize Lorentz non-invariant interactions. At dimension five level the experimental sensitivity for most is better than $10^{-5} M_{Pl}^{-1}$. If there are preferred frames, there is a possibility for CPT-odd EDMs.

3. Bekenstein-type models can accommodate $\Delta \alpha / \alpha \sim \mathcal{O}(1)$ only if the light field that "renormalizes" the coupling is cosmologically driven by dark matter or self-potential, not baryons.

4. The theoretical status of models with extra space-time anisotropy and/or changing couplings is difficult. There are no known UV complete theories that lead to condensation of a tensor. There is a sever naturalness problem for interacting quintessence.

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