Overview of the Standard-Model value of $a_{\mu}$ - especially the hadronic contribution.

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4th International Symposium on LEPTON MOMENTS,
Cape Cod, MA, July 19, 2010

Abstract
On status and prospects of calculating the muon anomalous magnetic moment, emphasizing the problems with including the hadronic effects.
Outline of Talk:

- The Anomalous Magnetic Moment of the Muon: Status
- Improvements due to the new $\pi^+\pi^-$ data from BaBar and KLOE
- About the hadronic light-by-light scattering contribution
- Comment on $e^+e^-$ versus $\tau$-data
- Summary and Outlook
The muon anomalous magnetic moment

Spinning particles have a magnetic moment: for the muon

\[ \vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c} \vec{s} \ ; \ g_\mu = 2 \ (1 + a_\mu) \]

Dirac: \( g_\mu = 2 \), \( a_\mu \) muon anomaly

Stern, Gerlach 22: \( g_e = 2 \); Kusch, Foley 48: \( g_e = 2 \ (1.00119 \pm 0.00005) \)

\[ \gamma(q) \overset{\mu(p')}{\sim} \mu(p) \]

\[ = (-ie) \bar{u}(p') \left[ \gamma^\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_2(q^2) \right] u(p) \]

\[ F_1(0) = 1 \ ; \ F_2(0) = a_\mu \]
Schwinger 1948: $a_e > 0$ explained by $O(\alpha)$ QED contribution:

$$a_e = \frac{\alpha}{2\pi} = 1.16 \times 10^{-3}$$

$\Rightarrow a_e$ sensitive to quantum field effects

universal for all leptons.

$\square$ $a_\mu$ responsible for the Larmor precession

$$\tilde{\omega}_a = \frac{e}{m} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right] E \sim 3.1 \text{ GeV}$$

$\Rightarrow$ directly proportional at magic energy $\sim 3.1$ GeV

Basic principle of experiment: measure Larmor precession of highly polarized muons circulating in a ring

CERN, BNL g-2 experiments
BNL Result and Update

$\alpha_\mu$ measured via a ratio of frequencies (measurement of $\alpha_\mu$ and $B$)

$$\omega_a = \omega_{\text{precession}} - \omega_{\text{cyclotron}} = \alpha_\mu \frac{eB}{m_\mu}; \quad \omega_{\text{precession}} = \omega_L + \omega_T; \quad \omega_{\text{cyclotron}} = \frac{e}{m_\mu} B$$

$$\alpha_\mu = \frac{\omega_a}{\omega_L - \omega_a} = \frac{\omega_a/\tilde{\omega}_p}{\omega_L/\tilde{\omega}_p - \omega_a/\tilde{\omega}_p} = \frac{R}{\lambda - R}$$

- $\tilde{\omega}_p = \frac{e}{m_\mu} \langle B \rangle$ free proton NMR frequency
- $R = \omega_a/\tilde{\omega}_p$ from E-821
- $\lambda = \omega_L/\tilde{\omega}_p = \mu_\mu/\mu_p$ from hyperfine splitting of muonium

value used by E-821: 3.18334539(10)
new value: 3.183345137(85) Mohr et al. RMP 80 (2008) 633

$\Rightarrow$ change in $\alpha_\mu$: $+0.92 \times 10^{-10}$ review in RPP2009 Höcker, Marciano

$$a_{\mu}^{\exp} = (11659208.9 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10} \text{ updated}$$
The role of $a_\mu$ in precision physics

Precision measurement of $a_\mu$ provides most sensitive test of magnetic helicity flip transition

$$\bar{\psi}_L \sigma_{\mu\nu} F^{\mu\nu} \psi_R \quad (\text{dim}_5 \text{ operator})$$

such a term must be absent for any fermion in any renormalizable theory at tree level

$\downarrow$

$a_\mu$ is a pure “quantum correction” effect:

a finite model-specific prediction

in any renormalizable quantum field theory (QFT)

☐ – test of quantum structure

☐ – monitor for new physics
\[ a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{weak}} + a_\mu^{\text{beyondSM}} \]

- $a_\mu^{\text{QED}}$: QED up to $O(\alpha^4)$ plus leading 5 loops; 5 loops in progress Kinoshita et al., Aoyama et al.

- $a_\mu^{\text{had}}$

- $a_\mu^{\text{weak}}$: weak SM one and two-loops complete Czarnecki, Marciano, ..., Stöckinger

- $a_\mu^{\text{beyondSM}}$

Highly complex mathematics meets reality!
Standard Model Prediction for $a_\mu$

1 QED Contribution

$a_\mu^{\text{QED}}$ has been computed at $O(\alpha^4)$ including leading 5 loops terms.

Growing coefficients in the $\alpha/\pi$ expansion reflect the presence of large $\ln\frac{m_\mu}{m_e} \approx 5.3$ terms coming from electron loops.

$$a_e^{\text{exp}} = 0.001\,159\,652\,180\,73(28)$$ Gabrielse et al. 2008

$\alpha^{-1}(a_e) = 137.035999084(51)[0.37\text{ppb}]$

based on work of Kinoshita et al.

$$a_\mu^{\text{QED}} = 116\,584\,718.564 \begin{bmatrix} \alpha_{\text{inp}} \\ m_e/m_\mu \\ \alpha^4 \\ \alpha^5 \end{bmatrix} \times 10^{-11}$$

The current uncertainty is well below the $\pm 60 \times 10^{-11}$ experimental error from E821
<table>
<thead>
<tr>
<th># of loops</th>
<th>( C_i [(\alpha/\pi)^n] )</th>
<th>( a_\mu^{\text{QED}} \times 10^{11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+0.5</td>
<td>116140973.289 (43)</td>
</tr>
<tr>
<td>2</td>
<td>+0.765 857 410(26)</td>
<td>413217.620 (14)</td>
</tr>
<tr>
<td>3</td>
<td>+24.050 512 28(46)</td>
<td>30141.905 (1)</td>
</tr>
<tr>
<td>4</td>
<td>+130.8105(85)</td>
<td>380.807 (25)</td>
</tr>
<tr>
<td>5</td>
<td>+731.0(1.1)</td>
<td>4.943 (8)</td>
</tr>
<tr>
<td>tot</td>
<td></td>
<td>116584718.564 (0.054)</td>
</tr>
</tbody>
</table>

1 diagram

Schwinger 1948

2 7 diagrams

Peterman 1957, Sommerfield 1957

3 72 diagrams

Lautrup, Peterman, de Rafael 1974, Laporta, Remiddi 1996

4 about 1000 diagrams


5 estimate of leading terms

Karshenboim 93, Czarnecki, Marciano 00

attack at full calculation:

Kinoshita, Nio 05, Ayoama et al. 09
Weak Contribution

Brodsky, Sullivan 67, ..., Bardeen, Gastmans, Lautrup 72
Higgs contribution tiny!
\[ a_{\mu}^{\text{weak}(1)} = (194.82 \pm 0.02) \times 10^{-11} \]

Kukhto et al 92
potentially large terms \[ \sim G_F m_{\mu\pi}^2 \alpha \ln \frac{M_Z}{m_{\mu}} \]
Peris, Perrottet, de Rafael 95
quark-lepton (triangle anomaly) cancel
Czarnecki, Krause, Marciano 96

final full 2-loop: Heinemeyer, Stöckinger, Weiglein 04, Gribouk, Czarnecki 05
\[ a_{\mu}^{\text{weak}(2)} = (-42.08 \pm 1.5[m_H, m_t] \pm 1.0[\text{had}]) \times 10^{-11} \]

Most recent evaluations: improved hadronic part (beyond QPM)
\[ a_{\mu}^{\text{weak}} = (153.2 \pm 1.0[\text{had}] \pm 1.5[m_H, m_t, 3 - \text{loop}]) \times 10^{-11} \]
(Knecht, Peris, Perrottet, de Rafael 02, Czarnecki, Marciano, Vainshtein 02)
Hadronic Contribution [limiting precision of theory]

(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$
(b) Hadronic light-by-light scattering $O(\alpha^3)$
(c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m^2_\mu)$

Light quark loops
↓

Hadronic “blob”

Hadronic vacuum polarization based on $e^+e^-$-annihilation data:

\[(689.4 \pm 4.0) \times 10^{-10}\] [Teubner et al.]
\[(690.3 \pm 5.3) \times 10^{-10}\] [FJ, FJ&Nyffeler]
\[(695.5 \pm 4.1) \times 10^{-10}\] [Davier et al.]
Differences between experiments [in common range] (examples):

- 4.8 between KLOE ’08 and SND ’06
- 8.5 between KLOE ’08 and BABAR ’09

Recent results for hadronic LbL:

\[(10.5 \pm 2.6) \times 10^{-10}\]  
\[(11.6 \pm 3.9) \times 10^{-10}\]

\(\pi^0, \eta, \eta'\)  
L.D.  
83(12) \times 10^{-11}

\(\pi^\pm, K^\pm\)  
L.D.  
\(-19(13) \times 10^{-11}\)

\(+62(3) \times 10^{-11}\)  
S.D.  
\(q = (u, d, s, ... )\)

Hadronic light–by–light scattering is dominated by \(\pi^0\) exchange in the odd parity channel, pion loops etc. at long distances (L.D.) and quark loops incl. hard gluonic corrections at short distances (S.D.)
Hadronic LbL scattering is dominated by non-perturbative physics

- Left: the spectrum of invariant $\gamma\gamma$ masses obtained with the Crystal Ball detector. The three rather pronounced spikes seen are the $\gamma\gamma \rightarrow$ pseudoscalar (PS) $\rightarrow$ $\gamma\gamma$ excitations: PS=$\pi^0$, $\eta$, $\eta'$

- Right: hadronic light–by–light scattering is dominated by $\pi^0$ exchange in the odd parity channel, pion loops etc. at long distances (L.D.) and quark loops incl. hard gluonic corrections at short distances (S.D.)
Theory vs experiment: do we see New Physics?

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value</th>
<th>Error</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED incl. 4-loops+LO 5-loops</td>
<td>11 658 471.86</td>
<td>0.01</td>
<td>Remiddi, Kinoshita ...</td>
</tr>
<tr>
<td>Leading hadronic vac. pol.</td>
<td>690.3</td>
<td>5.3</td>
<td>2009 update</td>
</tr>
<tr>
<td>Subleading hadronic vac. pol.</td>
<td>-10.0</td>
<td>0.2</td>
<td>2006 update</td>
</tr>
<tr>
<td>Hadronic light–by–light</td>
<td>11.6</td>
<td>3.9</td>
<td>new evaluation (J&amp;N)</td>
</tr>
<tr>
<td>Weak incl. 2-loops</td>
<td>15.32</td>
<td>0.22</td>
<td>CMV06</td>
</tr>
</tbody>
</table>

Theory | 11 659 179.0 | 6.5 | –
Experiment | 11 659 208.9 | 6.4 | BNL
Exp.- The. | 3.3 standard deviations | 29.9 | 9.0 | –

Standard model theory and experiment comparison [in units $10^{-10}$]
Calculations of vacuum polarization effects

General problem in electroweak precision physics:
contributions from hadrons (quark loops) at low energy scales
perturbative QCD (pQCD) calculations break down!

Leptons

Quarks

Reason for pQCD failure:
● Spontaneous breaking of chiral symmetry: Nambu-Goldstone bosons π’s etc.
● Strong interaction of hadrons (large effective QCD coupling at low scales)
Photon VP: low energy effective graphs a) and b) and high energy graph c)

How to calculate such effects?
Tools for non-perturbative hadronic effects

- use experimental data together with Dispersion Relations (DR) [sum rules]
- low energy effective Lagrangians; testable hadronic models
- non-perturbative calculation in lattice QCD (LQCD)

**Dispersion Relations**

Presently: the only first principles (model independent) tool which allows to determine non-perturbative effects like hadronic vacuum polarization by relating needed non-perturbative quantities to measured one's. Our special interest: LO hadronic contribution to $g - 2$: insertion of hadronic vacuum polarization
- **Causality** implies **analyticity** which implies a (subtracted) dispersion relation (version of Cauchy's theorem).

For photon self-energy function:

\[
\Pi'(q^2) - \Pi'(0) = \frac{q^2}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi'(s)}{s(s-q^2-i\varepsilon)}.
\]

- Knowledge of imaginary part determines the analytic function, in particular its real part.

- Analyticity is nothing but the momentum space version of causality of propagation in configuration space.

- Causality: output only after input (in QFT signals propagate in forward light cone only): input at \( t_0 \), response \( K(\tau = t - t_0) = \Theta(t - t_0) S(t - t_0) \) (assuming time translation invariance)

In Fourier space:
\[ \tilde{K}(\omega) = \int_{-\infty}^{+\infty} d\tau K(\tau) e^{i\omega \tau} = \int_{0}^{+\infty} d\tau K(\tau) e^{-\eta \tau} e^{i\xi \tau} \]

\[ \Rightarrow \tilde{K}(\omega = \xi + i\eta) \text{ is regular analytic function in the upper half } \omega-\text{plane } \eta > 0. \] This of course only works because \( \tau \) is restricted to be positive, which means causal.

In QFT: time ordered Green functions encode all information of the theory

- relativistic causality: particle (propagating forward) antiparticle (propagating backward) yields analyticity in entire energy plane

- analyticity is a general property of any QFT holding beyond perturbation theory! Key for applicability of lattice QCD!

- *Unitarity* implies the optical theorem:
slopy: $S$- martix unitary $S = 1 + i T$; $SS^+ = 1 \Rightarrow \langle f | SS^+ | i \rangle = \langle f | i \rangle$ for $|f\rangle = |i\rangle$
(forward scattering) means $2i \text{Im } T_{ii} \propto \sum_n |T_{in}|^2 \propto \sigma_{\text{tot}}$ (total cross section)

Optical theorem for scattering and propagation.

For the hadronic contribution to the photon propagator it reads

$$\text{Im } \Pi'_{\gamma, \text{had}}(s) = \frac{1}{12\pi} R(s); \quad R(s) \equiv \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})/\left(\frac{4\pi\alpha^2(s)}{3s}\right)$$

By definition $R(s)$ represents the inclusive hadronic cross section in units of the point cross section (tree level) $\sigma_{\mu\mu}(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)$ in the limit $s \gg 4m_{\mu}^2$, i.e. phase space mass effects corrected for.

Effective fine structure constant $\alpha_{\text{em}}(s)$:
Dressed photon propagator (modulo unphysical gauge dependent terms)

\[
-\frac{i e^2 g^{\mu\nu}}{q^2} \rightarrow -\frac{i e^2 g^{\mu\nu}}{q^2 (1 + \Pi'_\gamma(q^2))} + \text{gauge terms}
\]

interpretation: charge has to be replaced by a running charge

\[
e^2 \rightarrow e^2(q^2) = \frac{e^2 Z_\gamma}{1 + \Pi'_\gamma(q^2)} = \frac{e^2}{1 + (\Pi'_\gamma(q^2) - \Pi'_\gamma(0))}
\]

normalized to Thomson limit (photon wave function renormalization factor $Z_\gamma$ is fixed to get classical charge at $q^2 \rightarrow 0$). In terms of the fine structure constant $\alpha = \frac{e^2}{4\pi}$ reads

\[
\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha} \quad ; \quad \Delta\alpha = -\text{Re} \left( \Pi'_\gamma(q^2) - \Pi'_\gamma(0) \right).
\]
Contribution to $a_\mu$:

- standard evaluation of the non-perturbative hadronic contributions via DR in terms of measured cross-sections $\sigma(e^+e^- \to \text{hadrons})$:

$$a_{\mu, \text{LO}}^{\text{had}} = \frac{1}{4\pi^3} \int \frac{ds}{4m^2_\pi} K(s) \sigma^0_{\text{had}}(s) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int \frac{ds}{4m^2_\pi} \hat{K}(s) R^0_{\text{had}}(s) ,$$

with kernel function $\hat{K}(s) = \frac{3s}{m^2_\mu} K(s)$

$$K(s) = \frac{x^2}{2} (2 - x^2) + \frac{(1 + x^2)(1 + x)^2}{x^2} \left( \ln(1 + x) - x + \frac{x^2}{2} \right) + \frac{(1 + x)}{x^2} \ln(x) ,$$

where $x = (1 - \beta_\mu)/(1 + \beta_\mu)$, $\beta_\mu = \sqrt{1 - 4m^2_\mu/s}$ and undressed cross-section

$$\sigma^0_{\text{had}}(s) = \sigma_{\text{had}}(s) \left(\frac{\alpha(0)}{\alpha(s)}\right)^2 .$$

At low $Q^2$: $\propto \int ds/s^4 \cdots$ dominated by $\gamma^* \to \pi^+\pi^-$ pion form factor $|F_\pi(s)|^2$

mainly CMD2/SND Novosibirsk, KLOE Frascati, new: BABAR at SLAC
Evaluation of $a_{\mu}^{\text{had}}$

Leading non-perturbative hadronic contributions $a_{\mu}^{\text{had}}$ can be obtained in terms of

$$R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \rightarrow \gamma \rightarrow \text{hadrons})/\frac{4\pi\alpha^2}{3s}$$
data via dispersion integral:

$$a_{\mu}^{\text{had}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left( \int_{E_\text{cut}^2}^{E_\text{cut}^2} ds \frac{R_{\gamma}^{\text{data}}(s) \hat{K}(s)}{s^2} + \int_{4m_\pi^2}^{\infty} ds \frac{R_{\gamma}^{\text{QCD}}(s) \hat{K}(s)}{s^2} \right)$$

- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 36\%$ of error on $a_{\mu}^{\text{had}}$ comes from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$,
- 60% from 1 to 2 GeV

$$a_{\mu}^{\text{had}(1)} = (690.3 \pm 5.3)[695.5 \pm 4.1] \times 10^{-10}$$

$e^+e^- - \text{data based [incl. BaBar MD09]}$
Progress in the Determination of the Hadronic Vacuum Polarization

\[ \gamma_{\text{hard}} \]

\[ s = M_{\phi}^2; \quad s' = s (1 - k), \quad k = E_{\gamma} / E_{\text{beam}} \]

- Major progress by radiative return [initial state radiation (ISR)] [a)]
  measurements from fixed energy machines: BaBar/Belle at SLAC/KEK
  \( B \)-factories and KLOE at Frascati \( \phi \)-factory, many (old and new) channels
  in problematic region 1.4 to 2.4 GeV

- adding substantially to usual energy scan [b)]
  data from Novosibirsk CMD-2/SND and Beijing BES.

H. Kühn’s talk
At low energies (< 2 GeV) only measurements of exclusive channels, two approaches:

Energy scan (CMD2, SND):

- energy of colliding beams is changed to the desired value
- “direct” measurement of cross sections
- needs dedicated accelerator/physics program
- needs to measure luminosity and beam energy for every data point

Radiative return (KLOE, BABAR, BELLE):

- runs at fixed-energy machines (meson factories)
- use initial state radiation process to access lower lying energies or resonances
- data come as by-product of standard physics program
- requires precise theoretical calculation of the radiator function
- luminosity and beam energy enter only once for all energy points
- needs larger integrated luminosity
$e^+e^-$ data: current and future activities

DAFNE-2: DAFNE upgraded in energy (up to 2-2.5 GeV) with a luminosity $\sim 10^{32}$ cm$^{-2}$s$^{-1}$ ($\sim 5$ pb$^{-1}$ per day $\Leftrightarrow$ $1$ fb$^{-1}$/year)

from G. Venanzoni
BaBar: New: final $e^+e^- \to \pi\pi\gamma$ data Aug 2009 $\pi\pi$-spectrum from one experiment in large energy range !!!

Compilation of $\pi\pi$–data including new BaBar data from radiative return measurement at the $\Upsilon(4S)$ resonance (Davier et al 2009).
Relative cross section comparison between individual experiments. Shown are BABAR (top left), KLOE (top right), CMD2 (bottom left) and SND (bottom right).
### Table

<table>
<thead>
<tr>
<th>$m_{\pi\pi}$ range (GeV)</th>
<th>$a_{\mu}^{\pi\pi(\gamma),LO}$ BABAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.28 – 0.30</td>
<td>0.55 ± 0.01 ± 0.01</td>
</tr>
<tr>
<td>0.30 – 0.50</td>
<td>57.62 ± 0.63 ± 0.55</td>
</tr>
<tr>
<td>0.50 – 1.00</td>
<td>445.94 ± 2.10 ± 2.51</td>
</tr>
<tr>
<td>1.00 – 1.80</td>
<td>9.97 ± 0.10 ± 0.09</td>
</tr>
<tr>
<td>0.28 – 1.80</td>
<td>514.09 ± 2.22 ± 3.21</td>
</tr>
</tbody>
</table>

### Figure

In units $10^{-10}$, 0.7% precision!

* Davier et al. arXiv:0906-5443
KLOE: New $e^+ e^- \rightarrow \pi \pi \gamma$ data Jun 2010 $\sigma^{\pi \pi}(s)$ in $0.1 < s < 0.85 \text{ GeV}^2$.

\[
\Delta a_{\mu}^{\pi \pi} = (478.5 \pm 2.0_{\text{sta}} \pm 4.8_{\text{sys}} \pm 2.9_{\text{the}}) \times 10^{-10}
\]

The new result gives a value of $\Delta a_{\mu}^{\pi \pi}$ which is lower by $(0.8 \pm 0.9)\%$:

<table>
<thead>
<tr>
<th></th>
<th>$\Delta a_{\mu}^{\pi \pi}(0.35 - 0.85 \text{ GeV}^2) \times 10^{-10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KLOE10 (New!)</td>
<td>$376.6 \pm 0.9_{\text{stat}} \pm 2.4_{\text{exp}} \pm 2.1_{\text{th}}$</td>
</tr>
<tr>
<td>KLOE08</td>
<td>$379.6 \pm 0.4_{\text{stat}} \pm 2.4_{\text{exp}} \pm 2.2_{\text{th}}$</td>
</tr>
</tbody>
</table>

The combined fractional total error of $\Delta a_{\mu}^{\pi \pi}$ in KLOE range is $1.0\%$:

$\Delta a_{\mu}^{\pi \pi}(0.1 - 0.95 \text{ GeV}^2) = (488.6 \pm 4.1_{\text{indep.}} \pm 2.9_{\text{common}}) \times 10^{-10}$.
Comparison: SND, CMD-2, BaBar, KLOE

Ambrosino et al. arXiv:1006-5313
\[ \Delta a_{\mu}^{\pi\pi}(0.35-0.85\text{GeV}^2): \]

\[
\Delta a_{\mu}^{\pi\pi} = \frac{1}{4\pi^3} \int_{s_{1}}^{s_{2}} \sigma^{\text{ex}}(s) K(s) ds
\]

**KLOE08 (small angle)**

\[ a_{\mu}^{\pi\pi} = (379.6 \pm 0.4_{\text{stat}} \pm 2.4_{\text{sys}} \pm 2.2_{\text{theo}}) \cdot 10^{-10} \]

**KLOE09 (large angle)**

\[ a_{\mu}^{\pi\pi} = (376.6 \pm 0.9_{\text{stat}} \pm 2.4_{\text{sys}} \pm 2.1_{\text{theo}}) \cdot 10^{-10} \]

0.2% 0.6% 0.6%
\[ \Delta a_{\mu, \pi\pi} \text{ for different exp.}: \]

\[ \Delta a_{\mu, \pi\pi}(0.35-0.85 \text{GeV}^2): \]

\[ a_{\mu, \pi\pi} = \frac{1}{4\pi^3} \int_{s_1}^{s_2} \sigma^\text{had}(s) K(s) \, ds \]

**KLOE08 (small angle)**

\[ a_{\mu, \pi\pi} = (379.6 \pm 0.4_{\text{stat}} \pm 2.4_{\text{sys}} \pm 2.2_{\text{theo}}) \cdot 10^{-10} \]

**KLOE09 (large angle)**

\[ a_{\mu, \pi\pi} = (376.6 \pm 0.9_{\text{stat}} \pm 2.4_{\text{sys}} \pm 2.1_{\text{theo}}) \cdot 10^{-10} \]

**KLOE09 (large angle)**

\[ a_{\mu, \pi\pi} = (48.1 \pm 1.2_{\text{stat}} \pm 1.2_{\text{sys}} \pm 0.4_{\text{theo}}) \cdot 10^{-10} \]

**CMD-2**

\[ a_{\mu, \pi\pi} = (46.2 \pm 1.0_{\text{stat}} \pm 0.3_{\text{sys}}) \cdot 10^{-10} \]
\[ \Delta a_{\mu}^{\pi\pi} \text{ for different exp.:} \]

\[ a_{\mu}^{\pi\pi} = \frac{1}{4\pi^3} \int_{s_1}^{s_2} \sigma^{\text{had}}(s) K(s) \, ds \]

**\( \Delta a_{\mu}^{\pi\pi}(0.35-0.85 \text{GeV}^2) \):**

- **KLOE08 (small angle)**
  \[ a_{\mu}^{\pi\pi} = (379.6 \pm 0.4_{\text{stat}} \pm 2.4_{\text{sys}} \pm 2.2_{\text{theo}}) \cdot 10^{-10} \]

- **KLOE09 (large angle)**
  \[ a_{\mu}^{\pi\pi} = (376.6 \pm 0.9_{\text{stat}} \pm 2.4_{\text{sys}} \pm 2.1_{\text{theo}}) \cdot 10^{-10} \]

**\( \Delta a_{\mu}^{\pi\pi}(0.152-0.270 \text{GeV}^2) \):**

- **KLOE09 (large angle)**
  \[ a_{\mu}^{\pi\pi} = (48.1 \pm 1.2_{\text{stat}} \pm 1.2_{\text{sys}} \pm 0.4_{\text{theo}}) \cdot 10^{-10} \]

- **CMD-2**
  \[ a_{\mu}^{\pi\pi} = (46.2 \pm 1.0_{\text{stat}} \pm 0.3_{\text{sys}}) \cdot 10^{-10} \]

**\( \Delta a_{\mu}^{\pi\pi}(0.397-0.918 \text{GeV}^2) \):**

- **KLOE08 (small angle)**
  \[ a_{\mu}^{\pi\pi} = (356.7 \pm 0.4_{\text{stat}} \pm 3.1_{\text{sys}}) \cdot 10^{-10} \]

- **CMD-2**
  \[ a_{\mu}^{\pi\pi} = (361.5 \pm 1.7_{\text{stat}} \pm 2.9_{\text{sys}}) \cdot 10^{-10} \]

- **SND**
  \[ a_{\mu}^{\pi\pi} = (361.0 \pm 2.0_{\text{stat}} \pm 4.7_{\text{sys}}) \cdot 10^{-10} \]

- **BaBar**
  \[ a_{\mu}^{\pi\pi} = (365.2 \pm 1.9_{\text{stat}} \pm 1.9_{\text{sys}}) \cdot 10^{-10} \]
\[ a_\mu = \frac{(g_\mu - 2)}{2} \]

Theoretical predictions compared to the BNL result (2009)

- The latest inclusion of all e^e\text{-}data (DHMYZ09) gives a discrepancy btw \( a_\mu^{\text{SM}} \) and \( a_\mu^{\text{EXP}} \) of 3.2\sigma.

- Remaining differences on \( \sigma_{\pi\pi} \) btw different experiments (mainly KLOE/BaBar) to be clarified [\( \Delta a_\mu^{\text{EXP-SM}} = 2.4 + 3.7\sigma \)]

- (Reduced) discrepancy with \( \tau \) data (new l. corr., ee, \( \tau \) data) \[ a_\mu^{\text{exp}} - \Delta a_\mu^{\tau} = 1.4\sigma \]

KLOE09 is not yet in.

from G. Venanzoni
Conclusions

- KLOE has performed the first precision measurement of $\sigma_{\pi\pi}$ in the region 0.35 - 0.95 GeV$^2$ with ISR $\rightarrow$ 1.3% systematic error (KLOE05, *PLB 606, 12 (2005))
  - discrepancy between $a_{\mu}^{SM}$ and BNL experiment ($\sim 3\sigma$)

- KLOE has presented a new measurement in 2008 (KLOE08, *Phys. Lett. B 670, 285 (2009)*) with a different data sample using the same selection of KLOE05 (photon at small angle) $\rightarrow$ 0.9% systematic error
  
  • KLOE08 confirms the discrepancy of $\sim 3\sigma$ between $a_{\mu}^{SM}$ and $a_{\mu}^{EXP}$
  • KLOE08 $a_{\mu}^{\pi\pi}$ agrees with recent results from CMD2 and SND experiments. Reasonable agreement on $\sigma_{\pi\pi}$ shapes

- KLOE has presented a new measurement of $\sigma_{\pi\pi}$ in 2009 (KLOE09) in the range 0.1 - 0.85 GeV$^2$ using data taken at 1.0 GeV (20 MeV below the $\phi$-peak), with a different selection of KLOE08 $\rightarrow$ 1.0% systematic error
  
  • Very good agreement with KLOE08 in the overlapping region (0.35-0.85 GeV$^2$). Combination of the two measurements in progress
  • Agreement within errors with BaBar below 0.6 GeV; BaBar lies higher (2-3%) above

from G. Venanzoni
Outlook

- Measurement of $\sigma_{\pi\gamma}$ from $\pi\gamma/\mu\gamma$ ratio (as done by BaBar) well advanced.
  - Comparison of $\mu\gamma_{\text{DATA}}/\mu\gamma_{\text{MC}}$ will provide a consistency test for Radiator, Luminosity, FSR etc...
  - Results are expected for Summer conferences

- Check of FSR by Forward-Backward asymmetry (in progress)

- Still about 1.5 fb$^{-1}$ of KLOE from 2004/2005 data to be analyzed (3 times the statistics used up to now)

- Very important for $a_\mu$ also the region between 1 and 2 GeV. Already a lot has been done from BaBar and Belle with ISR, and more will come also from BES-III. To reach the ultimate precision of 1% projects like VEPP2000 and DAFNE-2 (DAFNE upgraded in energy) will be essential.

Stay Tuned!

from G. Venanzoni
Impact of DAFNE-2 on $(g-2)_\mu$

$$a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} = (27.7 \pm 8.4) \times 10^{-10} \quad (3.3\sigma)$$  

[Evdelman, TAU08]

8.4 = $5_{\text{HLO}} + 3_{\text{HLbL}} + 6_{\text{BNL}}$

4  2.6  2.5  1.6

DAFNE-2  NEW G-2

7-8σ (if 27.7 will remain the same)

$$\delta a_\mu^{\text{HLO}} = 3.3 (\sqrt{s} < 1\,\text{GeV}) \oplus 3.9 (1 < \sqrt{s} < 2\,\text{GeV}) \oplus 1.2 (\sqrt{s} > 2\,\text{GeV})$$

$$\delta a_\mu^{\text{HLO}} = 2.6 = 1.9 (\sqrt{s} < 1\,\text{GeV}) \oplus 1.3 (\sqrt{s} < 1\,\text{GeV}) \oplus 1.2 (\sqrt{s} > 2\,\text{GeV})$$

This means:

$$\delta \sigma_{\text{HAD}} \sim 0.4\% \sqrt{s} < 1\,\text{GeV} \quad \text{(instead of 0.7\% as now)}$$

$$\delta \sigma_{\text{HAD}} \sim 2\% \quad 1 < \sqrt{s} < 2\,\text{GeV} \quad \text{(instead of 6\% as now)}$$

With ISR at 1 GeV

With Energy Scan 1-2 GeV

Possible at DAFNE-2!

Precise measurement of $\sigma_{\text{HAD}}$ at low energies very important also for $\alpha_{\text{em}}(M_Z)$ (necessary for ILC) !!!

from G. Venanzoni
About the hadronic light-by-light scattering contribution

Hadrons in \( \langle 0 | T \{ A^\mu(x_1) A^\nu(x_2) A^\rho(x_3) A^\sigma(x_4) \} | 0 \rangle \)

Key object full rank-four hadronic vacuum polarization tensor

\[
\Pi_{\mu\nu\lambda\rho}(q_1, q_2, q_3) = \int d^4x_1 \ d^4x_2 \ d^4x_3 \ e^{i(q_1 x_1 + q_2 x_2 + q_3 x_3)} \\
\times \langle 0 | T\{ j_\mu(x_1) j_\nu(x_2) j_\lambda(x_3) j_\rho(0) \} | 0 \rangle.
\]

- non-perturbative physics

- fortunately, dominated by the pseudoscalar exchanges \( \pi^0, \eta, \eta', ... \) described by the effective Wess-Zumino Lagrangian

- generally, pQCD useful to evaluate the short distance (S.D.) tail
the dominant long distance (L.D.) part must be evaluated using some low energy effective model which includes the pseudoscalar Goldstone bosons as well as the vector mesons which play a dominant role (vector meson dominance mechanism).

QCD quark-loop result with current quark masses: 

\[ a_\mu^{(6)}(l,b,l, u + d) = 8229.34 \times 10^{-11} \]

and 

\[ a_\mu^{(6)}(l,b,l, s) = 17.22 \times 10^{-11} \]. Very sensitive to what quark mass should be used (current, constituent,...). Meaning of these result unclear.

Missing proper low energy structure of QCD. Need appropriate low energy effective theory ⇒ amount to calculate the following diagrams

LD contribution requires low energy effective hadronic models:
Resonance Lagrangian approach, incorporating VDM in accord with low energy structure of QCD (broken chiral symmetry, CHPT), like

\[ \mathcal{L}_{\text{int}}^{\text{HLS}} = -eg_\rho A_\mu \rho_\mu^0 - ig_\rho \pi \rho_\mu^0 (\pi^+ \overset{\rightarrow}{\partial} \mu \pi^-) - ig_\gamma \pi \pi A_\mu (\pi^+ \overset{\rightarrow}{\partial} \mu \pi^-) \\
+ (1 - a) e^2 A_\mu A_\mu \pi^+ \pi^- + 2eg_\rho \pi \pi A_\mu \rho_\mu^0 \pi^+ \pi^- - \frac{g_\rho}{F_\pi} A_\mu \left( V_{a_1}^+ \pi^- - V_{a_1}^- \pi^+ \right) \\
+ \cdots \]

Unfortunately, not unique! Should be singled out better by global fits. Based on such models, major efforts in estimating \( a_{\mu}^{\text{LbL}} \) were made by

- Hayakawa, Kinoshita, Sanda (HKS 1995), Hayakawa, Kinoshita (HK 1998) [HLS model]
- Bijnens, Pallante and Prades (BPP 1995) [ENJL model]
Problem: matching L.D. with S.D. ⇒ results depend on matching cut off \( \Lambda \) ⇒ model dependence (non-renormalizable low energy effective theory vs. renormalizable QCD tail)

Basic problem: \((s, s_1, s_2)\)–domain of \( \mathcal{F}_{\pi^0 \gamma^* \gamma^*}(s, s_1, s_2) \); here \((0, s_1, s_2)\)–plane

Two scale problem: “open regions”
- Data, OPE,
- QCD factorization,
- Brodsky-Lepage approach

One scale problem: “no problem”
- RLA
- pQCD

Novel approach: refer to quark–hadron duality of large-\( N_c \) QCD, hadron spectrum known, infinite series of narrow spin 1 resonances ‘t Hooft 79 ⇒ no matching problem (resonance representation has to match quark level representation)
De Rafael 94, Knecht, Nyffeler 02
Usually, adopting the narrow width approximation, one writes an amplitude

$$\text{Im} \ A(s) = \pi \sum g_i \delta(s - m_i^2) ; \quad \sum g_i = 1, \quad \sum g_i m_i^2 = 0, \quad \text{etc.}$$

where conditions on weight factors $g_i$ are required to reproduce constraints from chiral symmetry and high energy behavior (QCD, OPE).

Take a few of the lowest lying known resonances of appropriate quantum numbers and hope it to be a good approximation.

- **Knecht, Nyffeler** (KN 2001) [LMD+V model] discovered a sign mistake in the $\pi^0, \eta, \eta'$ exchange contribution, which changed the central value by $+167 \times 10^{-11}$! at that time.

- **Melnikov, Vainshtein** (MV 2004) [LMD+V model] found additional inconsistencies in previous calculations, this time in the short distance constraints (QCD/OPE) used in matching the high energy behavior of the effective models used for the $\pi^0, \eta, \eta'$ exchange contribution, shifts central value by $+53 \times 10^{-11}$!
So far all used on-shell pion pole approximation!

- General form–factor $F_{\pi^0\gamma^*\gamma^*}(s, s_1, s_2)$ is largely unknown

- The constant $e^2 F_{\pi^0\gamma\gamma}(m^2_{\pi}, 0, 0) = \frac{e^2 N_c}{12\pi^2 f_{\pi}} = \frac{\alpha}{\pi f_{\pi}} \approx 0.025 \text{ GeV}^{-1}$ well determined by $\pi^0 \rightarrow \gamma\gamma$ decay rate (from Wess-Zumino Lagrangian)

- Information on $F_{\pi^0\gamma^*\gamma^*}(m^2_{\pi}, -Q^2, 0)$ from $e^+e^- \rightarrow e^+e^-\pi^0$ experiments

**CELLO and CLEO measurement of the $\pi^0$ form factor $F_{\pi^0\gamma^*\gamma^*}(m^2_{\pi}, -Q^2, 0)$ at high space–like $Q^2$. outdated now by BaBar?**
Brodsky–Lepage interpolating formula gives an acceptable fit.

\[
\mathcal{F}_{\pi^0 \gamma^* \gamma}(m_{\pi^0}^2, -Q^2, 0) \approx \frac{1}{4\pi^2 f_{\pi}} \frac{1}{1 + (Q^2 / 8\pi^2 f_{\pi}^2)} \sim \frac{2f_{\pi}}{Q^2}
\]

Assuming the pole approximation this FF has been used by all authors (HKS,BPP,KN) in the past, but has been criticized recently (MV and FJ07).

- Melnikov, Vainshtein: in chiral limit vertex with external photon must be non-dressed! i.e. use \( \mathcal{F}_{\pi^0 \gamma^* \gamma}(0, 0, 0) \) which avoids kinematic inconsistency, thus no VDM damping \( \Rightarrow \) result increases by 30% !

- In \( g - 2 \) external photon at zero momentum \( \Rightarrow \) only \( \mathcal{F}_{\pi^0 \gamma^* \gamma}(0, -Q^2, 0) \) not \( \mathcal{F}_{\pi^0 \gamma^* \gamma}(m_{\pi^0}^2, -Q^2, 0) \) is consistent with kinematics. Unfortunately, this off–shell form factor is not known and in fact not measurable and CELLO/CLEO constraint does not apply! Obsolet far off-shell pion (in space-like region).

- Chiral limit in this case not a reasonable approximation!
Urgently need “model” for off–shell form–factor!

Back to Resonance Lagrangian Approach with cut off (seems more conservative)!

**Evaluation of $\alpha_{\mu}^{\text{LbL}}$ in the large-$N_c$ framework**

- Kneckt & Nyffeler and Melnikov & Vainshtein were using pion-pole approximation together with large-$N_c$ $\pi^0\gamma\gamma$–form-factor

- FJ & A. Nyffeler: relax from pole approximation, using KN off-shell LDM+V form-factor
\[ F_{\pi^0\gamma^*\gamma^*}(p_\pi^2, q_1^2, q_2^2) = \frac{F_\pi}{3} \frac{\mathcal{P}(q_1^2, q_2^2, p_\pi^2)}{Q(q_1^2, q_2^2)} \]

\[ \mathcal{P}(q_1^2, q_2^2, p_\pi^2) = h_7 + h_6 p_\pi^2 + h_5 (q_2^2 + q_1^2) + h_4 p_\pi^4 + h_3 (q_2^2 + q_1^2) p_\pi^2 \]

\[ + h_2 q_1^2 q_2^2 + h_1 (q_2^2 + q_1^2)^2 + q_1^2 q_2^2 (p_\pi^2 + q_2^2 + q_1^2) \]

\[ Q(q_1^2, q_2^2) = (q_1^2 - M_1^2) (q_1^2 - M_2^2) (q_2^2 - M_1^2) (q_2^2 - M_2^2) \]

all constants are constraint by SD expansion (OPE), except for \( h_3 + h_4 = 2 c_{VT} \)
with \( c_{VT} = M_{V_1}^2 M_{V_2}^2 \chi/2 \) and \( \Pi_{VT}(0) = -(\langle \bar{\psi}\psi \rangle_0 )/2\chi \) with evaluations of \( \chi[\text{GeV}^{-2}] \)

\( \chi[\text{GeV}^{-2}] = -2.7 \) (Ball et al. ’03) -3.3 (LMD) -8.2 (Ioffe & Smilga ’84) -8.9 (Vainshtein ’03)

First off-shell calculations:
FJ 08/Frascati: \( h_6 = (-5 \pm 5 \text{ GeV}^4 \) (positivity \( \Rightarrow \) WZW bounded, QPM), stability
Nyffeler 09 : \( h_6 = (+5 \pm 5 \text{ GeV}^4 \) (LDM vs LDM+V smoothness)

My own calculation: \( h_3 \in [-10, 10] \text{ GeV}^{-2} \)
<table>
<thead>
<tr>
<th>Contribution</th>
<th>BPP</th>
<th>HKS</th>
<th>KN</th>
<th>MV</th>
<th>PdRV</th>
<th>N/JN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0, \eta, \eta'$</td>
<td>85±13</td>
<td>82.7±6.4</td>
<td>83±12</td>
<td>114±10</td>
<td>114±13</td>
<td>99±16</td>
</tr>
<tr>
<td>$\pi, K$ loops</td>
<td>-19±13</td>
<td>-4.5±8.1</td>
<td>-</td>
<td>0±10</td>
<td>-19±19</td>
<td>-19±13</td>
</tr>
<tr>
<td>axial vectors</td>
<td>2.5±1.0</td>
<td>1.7±1.7</td>
<td>-</td>
<td>22±5</td>
<td>15±10</td>
<td>22±5</td>
</tr>
<tr>
<td>scalars</td>
<td>-6.8±2.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-7±7</td>
<td>-7±2</td>
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<td>quark loops</td>
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<td>9.7±11.1</td>
<td>-</td>
<td>-</td>
<td>2.3</td>
<td>21±3</td>
</tr>
<tr>
<td>total</td>
<td>83±32</td>
<td>89.6±15.4</td>
<td>80±40</td>
<td>136±25</td>
<td>105±26</td>
<td>116±39</td>
</tr>
</tbody>
</table>

\[ a_{\mu}^{\text{LbL;had}} = (116 \pm 39) \times 10^{-11} \]
Note: MV and KN utilize the same model LMD+V form factor:

\[
F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{4\pi^2 F^2_\pi}{N_c} \frac{q_1^2 q_2^2 (q_1^2 + q_2^2) - h_2 q_1^2 q_2^2 + h_5 (q_1^2 + q_2^2) + (N_c M_1^4 M_2^4 / 4\pi^2 F^2_\pi)}{(q_1^2 + M_1^2)(q_1^2 + M_2^2)(q_2^2 + M_1^2)(q_2^2 + M_2^2)},
\]

where \( M_1 = 769 \text{ MeV} \), \( M_2 = 1465 \text{ MeV} \), \( h_5 = 6.93 \text{ GeV}^4 \).

with two modifications:

- form factor \( F_{\pi^0\gamma^*\gamma^*}(q_2^2, q_2^2, 0) = 1 \): undressed soft photon (non-renormalization of ABJ) Note: to have anomaly correct does not imply that there is no damping! \( PVV \) anomaly quark loop is counter example; it has correct \( \pi\gamma\gamma \) in chiral limit (anomaly) and goes like \( m_q^2 / q_i^2 \) up to logs in all directions (but what \( m_q \)?)

- \( F_{\pi^0\gamma^*\gamma^*}(q_2^2, q_1^2, q_3^2) \approx F_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_3^2) = \text{KN} \)
  with \( h_2 = 0 \pm 20 \text{ GeV}^2 \) (KN) vs. \( h_2 = -10 \text{ GeV}^2 \) (MV) fixed by twist 4 in OPE \( (1 / q^4) \)

- \( a_1[f_1, f_1^*] \) different mixing scheme
Criticism: KN Ansatz only covers \((0, q_1^2, q_2^2)\)–plane, with consistent kinematics depends on 3 variables → 2–dim integral representation no longer valid.

Is this the final answer? How to improve? A limitation to more precise \(g - 2\) tests?

Looking for new ideas to get ride of model dependence

- Need better constained effective resonance Lagrangian (e.g. HSL and ENJL models vs. RLA of Ecker et al.). “Global effort” needed!

- Lattice QCD will provide an answer [far future (“yellow” region only)]!

- Try exploiting possible new experimental constraints:
\( \pi^0 \gamma \gamma \) form-factor: experimental possibilities

- time-like \((q_{\pi}^2 > 0)\) phenomenology (single tag data) versus space-like \((q_{\pi}^2 < 0)\) phenomenology poorly investigated, Primakoff-effect (\(\pi^0\) production by high energetic photons in Coulomb field of atomic nuclei)
The relation between the off-shell (needed for $a_\mu$) and the on-shell (measured) form-factor is not clear.

Existing data for $F(m_\pi^2, Q^2, 0): e^+e^- \rightarrow e^+e^-\pi^0$ single tag data $\frac{d\sigma}{dQ^2}$:

- CELLO: $0.5 \text{ GeV}^2 < Q^2 < 2.17 \text{ GeV}^2$  
  \cite{Z. Phys. C49 (1991) 401}

- CLEO: $1.5 \text{ GeV}^2 < Q^2 < 9 \text{ GeV}^2$  
  \cite{Phys. Rev. D57 (1998) 33}

- BABAR: $4 \text{ GeV}^2 < t_2 < 40 \text{ GeV}^2$  
  \cite{Phys. Rev. D80 (2009) 052002}

- new quest for theory

Before BaBar: consensus about large $Q^2$ behavior; $\pi^0$, $\eta$ and $\eta'$ consistent

Brodsky-Lepage (BL) $\sim 1/Q^2$

With BaBar: goes to higher $Q^2$ $\rightarrow$ violating BL behavior

BaBar: $\pi^0$, $\eta$ and $\eta'$ not consistent
asymptotic behavior is not understood ??? data consistent ???
$Q^2 F_{\pi^0\gamma^*\gamma}(m^2_{\pi}, Q^2, 0)$

Theory:
- [A. Nyffeler, 0912.1441]
- [M. Knecht and A. Nyffeler, Phys. Rev. D65, 073034 (2002)]
- [ibid.]
- [A. E. Dorokhov, 0905.4577]

No data at $0.02 \text{ GeV}^2 < Q^2 < 0.4 \text{ GeV}^2$
$\gamma^* \gamma^* \pi^0$ at KLOE-2

KLOE-2 experiment (PROJECT)

The $\phi(1020)$ meson factory DAΦNE (Frascati) + KLOE detector + small angle taggers

Sergiy IVASHYN (Katowica, Kharkov)  $\pi^0 \gamma \gamma$  21 / VI / 2010 @ Mainz  28 / 66
Tagging:
- single tagging LET: tagged invariant $t_1$ close to zero, promising range $0.05 \text{ GeV}^2 < t_2 < 0.4 \text{ GeV}^2$
- LET-LET and LET-HET double tagging is not possible
- LET + central: promising range $0.18 \text{ GeV}^2 < t_2 < 0.4 \text{ GeV}^2$
- single tagging HET: tagged invariant $t_1$ close to zero $\Rightarrow t_2$ also close to zero
- HET-HET double tagging is possible but both photons quasi-real $\Rightarrow$ good for measurement of $\pi^0 \rightarrow \gamma\gamma$ width, pion practically at rest
Single-tag measurement at BES-III
e.g. for extraction of the form-factor $F(m^{2}_\pi, Q^{2}, 0)$

Cross check of BABAR only possible by Belle!
Present and Future

\[ a_{\mu}^{\text{Exp.}} = 1.16592089(63) \times 10^{-3} \quad a_{\mu}^{\text{The.}} = 1.16591790(65) \times 10^{-3} \]

\[ \delta a_{\mu}^{\text{NP?}} = a_{\mu}^{\text{Exp.}} - a_{\mu}^{\text{The.}} = (299 \pm 90) \times 10^{-11} , \]

3.3 \sigma

Uncertainties:
- experiment: 0.54ppm = 6.3 \times 10^{-10}
- theory: 0.57ppm = 6.5 \times 10^{-10}
- new BNL 969 prop.: 0.20ppm = 2.4 \times 10^{-10}

What is it?
- statistical fluctuation of experimental result
- underestimated systematic error
- missing higher-order SM contributions
- underestimated theory error (incl. possible computational)
- physics beyond SM
Given theory results only differ by \( a_\mu^{\text{had}(1)} \)!
The Future

- We are hoping for follow up experiment at Fermilab (and/or JPAC/Japan)

- Improved hadronic VP needed (dominating present theory error)
  - Experimental program must go on: VEPP-2000, DAFNE-2, BES III
  - Lattice QCD will provide results within a few years to cross check and hopefully improve hadronic VP calculations up to 2 GeV

- Hadronic LbL the touchstone for theory:
  - Progress possible: sort out “the” realistic resonance Lagrangian [as the true low energy effective version of QCD] by global fit strategies
  - Lattice QCD calculations can provide in steps important results to cross check model calculations [very long term project]
- Independent check of 4–loop QED contribution highly desirable
- Progress in calculating 5–loop QED important for future progress
Further reading:

Book: F. Jegerlehner,
The Anomalous Magnetic Moment of the Muon,
Springer Tracts in Modern Physics,
Vol. 226, November 2007


Additional data: $\tau$–data + CVC

- $\tau^+ \gamma \tau^-$
- $e^+ \gamma e^-$
- $\bar{u}, \bar{d}$
- $u, d$
- $\pi^+, \pi^-, \cdots [I = 1]$

Isospin rotation

$\bar{\nu}_\mu W \tau^-$

$\pi^0, \pi^-, \cdots$

$\bar{u}, \bar{d}$
\[ \tau^- \rightarrow X^- \nu_\tau \iff e^+e^- \rightarrow X^0 \]

where \( X^- \) and \( X^0 \) are hadronic states related by isospin rotation. The \( e^+e^- \) cross-section is then given by

\[
\sigma^{I=1}_{e^+e^-\rightarrow X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-} , \quad \sqrt{s} \leq M_\tau
\]

in terms of the \( \tau \) spectral function \( v_1 \).

- mainly improves the knowledge of the \( \pi^+\pi^- \) channel (\( \rho \)-resonance contribution)
- which is dominating in \( \alpha^\text{had}_\mu \) (72%)
\( I = 1 \sim 75\% ; \ I = 0 \sim 25\% \)

t–data cannot replace \( e^+ e^- \)–data

\[
\begin{align*}
\delta a_\mu & : \ 15.6 \times 10^{-10} \ \rightarrow \ 10.2 \times 10^{-10} \\
\delta \Delta \alpha & : \ 0.00067 \ \rightarrow \ 0.00065 \quad (ADH1997)
\end{align*}
\]
Most recent measurement from Belle (2008):

\[ |F_p|^2 \]

\[
\begin{array}{c}
\text{Belle} \\
\text{ALEPH} \\
\text{CLEO} \\
\text{G\&S Fit}
\end{array}
\]

\[
(r_{770} + r_{1450} + r_{1700})
\]
$e^+e^- - \text{data}^* = \text{data corrected for isospin violations:}$

In $e^+e^-$ (neutral channel) $\rho - \omega$ mixing due isospin violation be quark mass difference $m_u \neq m_u \Rightarrow I=0$ component; to be subtracted for comparison with $\tau$ data

$$|F(s)|^2 = \left( |F(s)|^2 - \text{data} \right) / \left| 1 + \frac{\epsilon s}{(s_\omega - s)} \right|^2 \quad \text{with} \quad s_\omega = (M_\omega - \frac{i}{2} \Gamma_\omega)^2$$

$\epsilon$ determined by fit to the data: $\epsilon = 0.00172$

CMD-2 data for $|F_\pi|^2$ in $\rho - \omega$ region together with Gounaris-Sakurai fit. Left before subtraction right after subtraction of the $\omega$.

$I=0$ component to be added to $\tau$ data for calculating $a^\text{had}_\mu$!
**Other isospin-breaking corrections** Cirigliano et al. 2002, López Castro et al. 2007

**Left:** Isospin-breaking corrections $G_{EM}$, FSR, $\beta_0^3(s)/\beta_s^3(s)$ and $|F_0(s)/F_-(s)|^2$.
**Right:** Isospin-breaking corrections in $I = 1$ part of ratio $|F_0(s)/F_-(s)|^2$: $\pi$ mass splitting $\delta m_\pi = m_{\pi^\pm} - m_{\pi^0}$, $\rho$ mass splitting $\delta m_\rho = m_{\rho^\pm} - m_{\rho^0_{\text{bare}}}$, and $\rho$ width splitting $\delta \Gamma_\rho$. 
New isospin corrections applied shift in mass and width [as advocated by S. Ghozzi and FJ in 2003!!!] plus changes [López Castro, Toledo Sánchez et al 2007] below the \( \rho \) which Davier et al say are not understood! The discrepancy now substantially reduced but with the KLOE data persists. New BaBar radiative return \( \pi\pi \) spectrum in much better agreement, in particular with Belle \( \tau \) spectrum! What’s the truth?

\[ \frac{|F_{ee}|^2}{|F_\tau|^2} - 1 \] as a function of \( s \). Isospin-breaking (IB) corrections are applied to \( \tau \) data with its uncertainties included in the error band.
Revisited Analysis using $\tau$ Data: including Belle

Test of the spectral function shapes from different experiments: WA BR used

M. Davier  HVP/g-2  Physics LHC Era  21/3/2010
CVC prediction of $\mathcal{B}_{\pi\pi^0}$ normalization of BELLE, CLEO and OPAL not fixed by the experiment itself

The measured branching fractions for $\tau^- \to \pi^-\pi^0\nu_\tau$ compared to the predictions from the $e^+e^- \to \pi^+\pi^-$ spectral functions (after isospin-breaking corrections). (Named $e^+e^-$ results for $0.63 - 0.958\text{GeV}$). The long and short vertical error bands correspond to the $\tau$ and $e^+e^-$ averages of $25.42 \pm 0.10$ and $24.76 \pm 0.25$, respectively.

Note -2% in Belle $\tau$ data means $25.42 \to 24.91$ in agreement with $e^+e^-$ 

$\mid F_\tau(0) \mid^2 = 1.02 \to \mid F_\tau(0) \mid^2 = 1$
Possible origin of problems:

- Unknown isospin violations in parameters: $m_{\rho^+} - m_{\rho^0}$, $m_{\rho^0} - m_{\rho^0}$, same for widths, mixing parameters; largely not established (theor. and exper.)

Cottingham formula calculating $m_{\pi^-}^2 - m_{\pi^0}^2$ very successfully suggests

$$\Delta m_{\rho}^2 = \Delta m_{\pi}^2 \Rightarrow m_{\rho^+} - m_{\rho^0} \approx 0.81 \text{ MeV} \sim 1 \text{ MeV}$$

Also: $\Gamma_{\rho^0} = \left(\frac{m_{\rho^0}}{m_{\rho^-}}\right)^3 \left(\frac{\rho^0}{\beta^-}\right)^3 \Gamma_{\rho^-} + \Delta \Gamma_{\text{em}} \Rightarrow \Gamma_{\rho^-} - \Gamma_{\rho^0} \approx 2.1 \pm 0.5 \text{ MeV} \text{ radiative em corrections now included}$

- Needed what is measured in $e^+e^-$: $|A_{I=1}(s) + A_{I=0}(s)|^2 < |A_{I=1}(s)|^2 + |A_{I=0}(s)|^2$;
- $\tau$ evaluations based on $|A_{I=1}^\tau(s)|^2 + |A_{I=0}^{e^+e^-}(s)|^2$ which may overestimate the effects; separation of $|A_{I=0}^{e^+e^-}(s)|^2$ using Gounaris-Sakurai fit of the $\rho - \omega$ $[\varepsilon_{\rho \omega} = (2.02 \pm 0.1) \times 10^{-3}]$; (see HLS model calculation presented by Benayoun et al. which claims large diminution by interference).
hadronic final state photon radiation not under quantitative control, in \(\tau\)–decay enhanced short distance sensitivity (UV-log modeled by quark parton model, rest by sQED)

\[|F_\tau(s)|^2 \text{ yields } F_\tau(0) = 1.02 \pm 0.01 \pm 0.04 \Rightarrow \text{ this violates em current conservation. Benayoun et al. 2009 suggest that normalization may be wrong } \rightarrow \text{ shift down data by 2%; actually with global shift by -4.5%} \]
perfect agreement with Novosibirsk $e^+e^-$ data (as a distribution). Is the main problem that ALEPH lies very high ???
Relative comparison between the combined $\tau$ (dark shaded) and $e^+e^-$ spectral functions (light shaded), normalized to the $e^+e^-$ result.

M. Davier et al. 2009
Future improvements feasible!

Experimental: new proposal for muon $g - 2$ at FNAL

The New $(g - 2)$ Experiment:
A proposal to Measure the Muon Anomalous Magnetic Moment to $\pm 0.14$ ppm Precision

Present Status:

- **Experimental uncertainty**: $63 \times 10^{-11}$ (0.54 ppm)
  - 0.46 ppm **statistical**
  - 0.28 ppm **systematics**
- **Theory uncertainty**: $51 \times 10^{-11}$ (0.44 ppm)

Present: $\delta a_{\mu}(\text{Exp. - The.}) = (295 \pm 81) \times 10^{-11}$
Experimental situation after new experiment:

- **Experimental uncertainty:** $16 \times 10^{-11}$ (0.14 ppm)
  - 0.10 ppm statistical
  - 0.10 ppm systematics

- **Theory uncertainty:** $30 \times 10^{-11}$ (0.26 ppm)

**Future:** $\delta a_\mu(\text{Exp.} - \text{The.}) = (x x \pm 34) \times 10^{-11}$

If central values remain as now: present deviation $\Rightarrow 9\sigma$!

If SUSY or 2HDM most sensitive $\tan \beta$ monitor!

Stöckinger
Outlook

LHC the big challenge ahead!

- it's running!

Precision experiments remain an important complement of LHC:

- $a_\mu$ maybe the best!

- this we hope will be realized!

Time horizon for next step in improvement: 10 years

Will provide important information on Physics Beyond the SM scenarios!
Provided deviation is real $3\sigma \rightarrow 9\sigma$ possible?

If SUSY:

$$\delta a_\mu \leftrightarrow \text{sign}(\mu) \text{ and } \tan \beta$$

If not SUSY or 2HDM may be even more interesting!

In any case establishing a new theory replacing SM likely is a long way to go and requires efforts on very different levels

- So we hope muon $(g - 2)$ has a bright future, as we hope for the LHC!
- All challenging physics for our young colleagues in experiment and theory.
† Dispersion Relations

- **Causality** implies *analyticity* which implies a (subtracted) *dispersion relation* (version of Cauchy’s theorem).

For photon self-energy function:

\[
\Pi'_\gamma(q^2) - \Pi'_\gamma(0) = \frac{q^2}{\pi} \int_0^\infty ds \frac{\text{Im}\Pi'_\gamma(s)}{s(s^2-q^2-i\varepsilon)}.
\]

**Digression: the Origin of Analyticity**

- analyticity is real space causality in momentum space

- causality: output only after input (in QFT signals propagate in forward light cone only): input at \( t_0 \), response \( K(\tau = t - t_0) = \Theta(t - t_0) S(t - t_0) \) (assuming time translation invariance)

In Fourier space:
\[ \tilde{K}(\omega) = \int_{-\infty}^{+\infty} d\tau K(\tau) e^{i\omega \tau} = \int_{0}^{+\infty} d\tau K(\tau) e^{-\eta \tau} e^{i\xi \tau} \]

⇒ \( \tilde{K}(\omega = \xi + i\eta) \) is regular analytic function in the upper half \( \omega \)–plane \( \eta > 0 \).

This of course only works because \( \tau \) is restricted to be positive, which means causal.

In QFT: time ordered Green functions encode all information of the theory
in perturbation theory integrals over products of causal propagators 
\((z = x - y)\)

\[
iS_F(z) = \langle 0| T \{\psi(x)\bar{\psi}(y)\} |0\rangle \\
= \Theta(x^0 - y^0)\langle 0|\psi(x)\bar{\psi}(y)|0\rangle - \Theta(y^0 - x^0)\langle 0|\bar{\psi}(y)\psi(x)|0\rangle \\
= \Theta(z^0) iS^+(z) + \Theta(-z^0) iS^-(z)
\]

- positive frequency part propagating forward in time
- negative frequency part propagating backwards in time
- in QFT two terms needed; each has simple pole and half-plane analyticity
- together full-plane analyticity; allows for Wick rotation

Free Dirac field Feynman propagator in momentum space

\[
\tilde{S}_F(q) = \frac{q + m}{q^2 - m^2 + i\epsilon}
\]
analytic properties are manifest:

\[
\frac{1}{q^2 - m^2 + i\epsilon} = \frac{1}{q^0 - \sqrt{q^2 + m^2} - i\epsilon} - \frac{1}{q^0 + \sqrt{q^2 + m^2} - i\epsilon} = \frac{1}{2\omega_p} \left\{ \frac{1}{q^0 - \omega_p + i\epsilon} - \frac{1}{q^0 + \omega_p - i\epsilon} \right\}
\]

is an analytic function in \( q^0 \) with poles at \( q^0 = \pm (\omega_p - i\epsilon) \). It is the basis for the Wick rotation to the Euclidean region

\[
\frac{1}{q^2 - m^2 + i\epsilon} \rightarrow -\frac{1}{q^2 + m^2}
\]

Wick rotation in the complex \( q^0 \)-plane. The poles of the Feynman propagator are indicated by \( \otimes \)'s. \( C \) is an integration contour, \( R \) is the radius of the arcs.
Note: Causality is fundamental in any physical theory (predictability of the future)

- non-relativistic causality $\Rightarrow$ analyticity in half-plane

- relativistic causality particle (forward) antiparticle (backward) yields $\Rightarrow$ analyticity in entire energy plane

- only in relativistic theory Wick rotation is possible (equivalence Euclidean vs Minkowski).

- analyticity is a general property in any QFT and must hold beyond perturbation theory! Key for applicability of LQCD!

End of the digression.

- Unitarity implies the optical theorem:

For the hadronic contribution to the photon propagator it reads
By definition $R(s)$ represents the inclusive hadronic cross section in units of the point cross section (tree level) $\sigma_{\mu\mu}(e^+e^- \to \gamma^* \to \mu^+\mu^-)$ in the limit $s \gg 4m^2_\mu$, i.e. phase space mass effects corrected for.

Digression: the Optical Theorem (general: non-perturbative or perturbative)

unitarity in forward scattering

The optical theorem directly derives from unitarity of the $S$–matrix ($SS^+ = S^+S = 1$) translated to $T$–matrix elements ($S = 1 + i(2\pi)^4 \delta^{(4)}(P_f - P_i) T$):

$$i \left\{ T_{if}^* - T_{fi} \right\} = \oint_n (2\pi)^4 \delta^{(4)}(P_n - P_i) T_{nf}^* T_{ni} ,$$
in the limit of elastic forward scattering $|f\rangle \rightarrow |i\rangle$ where

$$2 \text{Im } T_{ii} = \oint_n (2\pi)^4 \delta^{(4)}(P_n - P_i) |T_{ni}|^2.$$ 

Graphically, this relation may be represented by (spectral decomposition)

Optical theorem for scattering and propagation.

It tells us that the imaginary part of the photon propagator is proportional to the total cross section $\sigma_{\text{tot}}(e^+e^- \rightarrow \gamma^* \rightarrow \text{anything})$ ("anything" means any...
possible state). The precise relationship reads

\[ \text{Im } \hat{\Pi}'(s) = \frac{1}{12\pi} R(s) \]

\[ \text{Im } \Pi'(s) = e(s)^2 \text{Im } \hat{\Pi}'(s) = \frac{s}{e(s)^2} \sigma_{\text{tot}}(e^+e^- \rightarrow \gamma^* \rightarrow \text{anything}) = \frac{\alpha(s)}{3} R(s) . \]

End of the digression.

\( e^+e^- \rightarrow \text{hadroms vs. photon self-energy} \)

Causality + unitarity \( \Rightarrow \) analyticity of \( \Pi'_\gamma(s) \) in complex \( s \)–plane with cut along the positive real axis starting at \( s \geq 4m_e^2 \).
Cauchy’s integral theorem: the contour integral satisfies

\[
F(s) = \frac{1}{2\pi i} \oint_C \frac{ds' F(s')}{s' - s}.
\]

Applied to \( F(s) = (\Pi'_\gamma(s) - \Pi'_\gamma(0))/s \) one can take \( R \to \infty \Rightarrow \) wanted subtracted (=renormalized) DR.

\( R \to \infty \Rightarrow \) wanted subtracted (=renormalized) DR.

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\[ R_{\text{had}}(s) = \sigma(e^+e^- \to \text{hadrons}) \left/ \frac{4\pi\alpha(s)^2}{3s} \right. \]

for leptons applies order by order in perturbation theory

for hadrons (quarks) on can use measured cross section \( e^+e^- \to \text{hadrons} \) to get
and the DR to obtain the relevant hadronic vacuum polarization

\[ \Pi'_{\gamma \text{ren}}(q^2) = \frac{\alpha q^2}{3\pi} \int_{4m^2_{\pi}}^{\infty} ds R_{\text{had}}(s) \frac{s}{s(s - q^2 - i\varepsilon)} . \]

**Effective fine structure constant** $\alpha_{\text{em}}(s)$:

**Dressed photon propagator (modulo unphysical gauge dependent terms)**

\[ i e^2 D_{\gamma}^{\mu\nu}(q) = -i \frac{e^2 g^{\mu\nu}}{q^2 (1 + \Pi'_{\gamma}(q^2))} + \text{gauge terms} , \]

**interpretation**: charge has to be replaced by a running charge

\[ e^2 \rightarrow e^2(q^2) = \frac{e^2 Z_{\gamma}}{1 + \Pi'_{\gamma}(q^2)} = \frac{e^2}{1 + (\Pi'_{\gamma}(q^2) - \Pi'_{\gamma}(0))} \]
normalized to *Thomson limit* (photon wave function renormalization factor $Z_\gamma$ is fixed to get classical charge at $q^2 \to 0$).

- lowest order diagram in perturbation theory

\[ \begin{array}{c}
\begin{array}{c}
\gamma \\
\downarrow \\
\uparrow \\
\gamma
\end{array}
\end{array} \]

and describes the virtual creation and re–absorption of fermion pairs

$\gamma^* \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, u\bar{u}, d\bar{d}, \ldots \to \gamma^*.$

In terms of the fine structure constant $\alpha = \frac{e^2}{4\pi}$ reads

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta \alpha}; \quad \Delta \alpha = -\text{Re} \left( \Pi'_\gamma(q^2) - \Pi'_\gamma(0) \right).$$

**Contribution to $a_\mu$:**

- standard evaluation of the non-perturbative hadronic contributions via DR in terms of measured cross-sections $\sigma(e^+e^- \to \text{hadrons})$: 
\[ a_{\mu}^{\text{had, LO}} = \frac{1}{4\pi^3} \int_{4m^2_\pi}^{\infty} ds \, K(s) \sigma^0_{\text{had}}(s) = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m^2_\pi}^{\infty} ds \, \frac{\hat{K}(s) R^0_{\text{had}}(s)}{s^2}, \]

with kernel function \( \hat{K}(s) = \frac{3s}{m_\mu^2} K(s) \)

\[ K(s) = \frac{x^2}{2} (2 - x^2) + \frac{(1 + x^2)(1 + x)^2}{x^2} \left( \ln(1 + x) - x + \frac{x^2}{2} \right) + \frac{(1 + x)}{(1 - x)} x^2 \ln(x), \]

where \( x = (1 - \beta_\mu)/(1 + \beta_\mu), \beta_\mu = \sqrt{1 - 4m^2_\mu/s} \) and undressed cross-section

\[ \sigma^0_{\text{had}}(s) = \sigma_{\text{had}}(s) \left( \frac{\alpha(0)}{\alpha(s)} \right)^2. \]

**At low \( Q^2: \propto \int ds/s^4 \cdots \text{dominated by } \gamma^* \rightarrow \pi^+\pi^- \text{ pion form factor } |F_\pi(s)|^2 \)**

mainly CMD2/SND Novosibirsk, KLOE Frascati, new: BABAR at SLAC
A new representation for single particle exchange in LbL

- $a_\mu$ does not depend on direction of muon momentum $p \Rightarrow$ may average in Euclidean space over the directions $\hat{P}$:

$$\langle \cdots \rangle = \frac{1}{2\pi^2} \int d\Omega(\hat{P}) \cdots$$

Hadronic single particle exchange amplitudes independent of $p \Rightarrow 2$ integrations may be done analytically: amplitudes $T_i$, propagators

(4) $\equiv (P + Q_1)^2 + m_\mu^2$ and (5) $\equiv (P - Q_2)^2 + m_\mu^2$ with $P^2 = -m_\mu^2$

$$\langle \frac{1}{(4)} \frac{1}{(5)} \rangle = \frac{1}{m_\mu^2 R_{12}} \arctan \left( \frac{zx}{1 - zt} \right)$$

$$\langle (P \cdot Q_1) \frac{1}{(5)} \rangle = -(Q_1 \cdot Q_2) \frac{(1 - R_{m2})^2}{8m_\mu^2}$$
\[ \langle (P \cdot Q_2) \frac{1}{4} \rangle = (Q_1 \cdot Q_2) \frac{(1 - R_{m1})^2}{8 m_\mu^2} \]
\[ \langle \frac{1}{4} \rangle = -\frac{1 - R_{m1}}{2 m_\mu^2} \]
\[ \langle \frac{1}{5} \rangle = -\frac{1 - R_{m2}}{2 m_\mu^2} \]

\[ R_{m_i} = \sqrt{1 + \frac{4 m_\mu^2}{Q_i^2}}, \quad (Q_1 \cdot Q_2) = Q_1 Q_2 t, \quad t = \cos \theta, \quad \theta = \text{angle between } Q_1 \text{ and } Q_2. \]

Denoting \( x = \sqrt{1 - t^2} \), we have \( R_{12} = Q_1 Q_2 x \) and
\[ z = \frac{Q_1 Q_2}{4 m_\mu^2} (1 - R_{m1}) (1 - R_{m2}) \].

- For any hadronic form-factor end up with 3–dimensional integral over
\( Q_1 = |Q_1|, \ Q_2 = |Q_2| \) and \( t = \cos \theta \):

\[
a_\mu(\text{LbL}; \pi^0) = -\frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 dQ_2 \int_{-1}^{+1} dt \ \sqrt{1 - t^2} \ Q_1^3 \ Q_2^3 \\
\times (F_1 \ P_6 \ I_1(Q_1, Q_2, t) + F_2 \ P_7 \ I_2(Q_1, Q_2, t))
\]

where \( P_6 = 1/(Q_2^2 + m_\pi^2) \), and \( P_7 = 1/(Q_3^2 + m_\pi^2) \) denote the Euclidean single particle exchange propagators. \( I_1 \) and \( I_2 \) known integration kernels. The non-perturbative factors are

\[
F_1 = \mathcal{F}_{\pi^0 \gamma^* \gamma^*}(q_2^2, q_1^2, q_3^2) \ \mathcal{F}_{\pi^0 \gamma^* \gamma^*}(q_2^2, q_2^2, 0), \quad F_2 = \mathcal{F}_{\pi^0 \gamma^* \gamma^*}(q_3^2, q_1^2, q_2^2) \ \mathcal{F}_{\pi^0 \gamma^* \gamma^*}(q_3^2, q_3^2, 0).
\]
Note: $SU(3)$ flavor decomposition of em current $\rightarrow$ weight factors

\[ W^{(a)} = \frac{\left( \text{Tr} [\lambda_a \hat{Q}^2] \right)^2}{\text{Tr} [\lambda_a^2] \text{Tr} [\hat{Q}^4]} ; \quad W^{(3)} = \frac{1}{4}, \quad W^{(8)} = \frac{1}{12}, \quad W^{(0)} = \frac{2}{3}. \]

where $\text{Tr} [\hat{Q}^4] = 2/9$ is the overall normalization such that $\sum_a W^{(a)} = 1$. Note $(W^{(8)} + W^{(0)})/W^{(3)} = 3$, higher states enhanced in coupling by factor 3! [Melnikov&Vainshtein] overlooked by previous analyzes [HKS,HK,BPP].
New Physics contributing to $a_\mu$

Possible New Physics contributions: neutral boson exchange: a) scalar or pseudoscalar and c) vector or axialvector, flavor changing or not, new charged bosons: b) scalars or pseudoscalars, d) vector or axialvector

(a) Case: $m_\mu = M \ll M_0$

(b) Case: $m_\mu \ll M_0 = M$
New physics typically:

\[ a^\text{NP}_\mu = C \frac{m^2_\mu}{M^2_{\text{NP}}} \]

where naturally \( C = O(\alpha/\pi) (~ \text{lowest order} ~ a^\text{SM}_\mu) \);

Typical New Physics scales required to satisfy \( \Delta a^\text{NP}_\mu = \delta a_\mu \):

<table>
<thead>
<tr>
<th>( C )</th>
<th>1</th>
<th>( \alpha/\pi )</th>
<th>( (\alpha/\pi)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{\text{NP}} )</td>
<td>( 2.0^{+0.4}_{-0.3} ) TeV</td>
<td>( 100^{+21}_{-13} ) GeV</td>
<td>( 5^{+1}_{-1} ) GeV</td>
</tr>
</tbody>
</table>

D. Stöckinger’s talk
The Muon $g - 2$

“the closer you look the more there is to see”

History of sensitivity to various contributions

The anomalous magnetic moment of the muon by itself a tiny 0.116 % effect now measured at $5 \times 10^{-7}$!