Measurement of Muon Dipole Moments

Gerco Onderwater
KVI, University of Groningen, the Netherlands

Lepton Moments 2010, Cape Cod, July 20 2010
Outline

- Basic Properties: $P$-violation, dipole moments
- Nevis era: signal definition
- CERN era: $\omega_a$, decay in flight, $p_{\text{magic}}$, B-field
- BNL era: putting it all together
- Future: FNAL, J-PARC, PSI, ...
Standard Model Properties

Second generation charged lepton

- Mass: $m_\mu \sim 206 \, m_e \sim 105 \, \text{MeV}/c^2$
- Charge: $q_\mu = q_e = 1e$
- Lifetime: $\tau_\mu \sim 2.2 \mu s$
- Production: $\pi^\pm \rightarrow \mu^\pm + \nu_\mu$
- Decay: $\mu^\pm \rightarrow e^\pm + \nu_\mu + \nu_e$

Weak interaction & parity violation

- Muons are produced fully polarized
- Decay electron distribution correlates with muon spin: asymmetry=1/3
EM Dipole Moments

This differs from (1) by the two extra terms

\[
\frac{e\hbar}{c} (\sigma, H) + \frac{ie\hbar}{c} \rho_1 (\sigma, E)
\]

in \( F \). These two terms, when divided by the factor \( 2m \), can be regarded as the additional potential energy of the electron due to its new degree of freedom.

\[
\vec{\mu} = g \frac{e \hbar}{2mc} \hat{\sigma} \quad \quad \vec{d} = \eta \frac{e \hbar}{2mc} \hat{\sigma}
\]

\[
g = 2(a + 1) \quad \quad \eta = 0
\]

Anomaly \quad P & T conservation

This is where the physics is!
Nevis
Stopped Muons
Larmor Precession

\[
\frac{d \vec{S}}{dt} = -\vec{\mu} \times \vec{B} \quad \rightarrow \quad \Omega = g \frac{e \hbar}{2mc} B
\]

With B measured in terms of proton NMR frequency

\[
g_{\mu} = g_p \frac{f_\mu}{f_p} \frac{m_\mu}{m_p}
\]

\[g_p = 2x(1.79275(3)+1)\]

Measure frequency ratio

206.76(25) m_e
1957: First experiment

Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon

Richard L. Garwin,† Leon M. Lederman, and Marcel Weinrich

Physics Department, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York, New York
(Received January 15, 1957)

\[ g = 2.0 \pm 0.1 \]
Results with stopped muons

<table>
<thead>
<tr>
<th>Year</th>
<th>$g_\mu$</th>
<th>$\sigma g_\mu$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>2.00</td>
<td>0.10</td>
<td>Garwin</td>
</tr>
<tr>
<td>1957</td>
<td>2.004</td>
<td>0.014</td>
<td>Cassels</td>
</tr>
<tr>
<td>1957</td>
<td>2.0064</td>
<td>0.0048</td>
<td>Coffin</td>
</tr>
<tr>
<td>1958</td>
<td>2.0030</td>
<td>0.0012</td>
<td>Lundy</td>
</tr>
<tr>
<td>1958</td>
<td>1.9986</td>
<td>0.0084</td>
<td>Lundy</td>
</tr>
<tr>
<td>1960</td>
<td>2.0024</td>
<td>0.0003</td>
<td>Garwin</td>
</tr>
</tbody>
</table>

Precision

$\Delta g/g = 150$ppm

$\Delta a/a = 11\%$

Muon behaves as heavy electron!
More on $\lambda = \mu_\mu / \mu_p$

<table>
<thead>
<tr>
<th>Year</th>
<th>$\lambda$</th>
<th>$\sigma_\lambda$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1963</td>
<td>3.18338</td>
<td>0.000004</td>
<td>Hutchinson</td>
</tr>
<tr>
<td>1969</td>
<td>3.183355</td>
<td>0.000048</td>
<td>Thompson; muonium HFS</td>
</tr>
<tr>
<td>1970</td>
<td>3.183330</td>
<td>0.000044</td>
<td>Hutchinson</td>
</tr>
<tr>
<td>1970</td>
<td>3.183373</td>
<td>0.000013</td>
<td>De Voe; muonium HFS</td>
</tr>
<tr>
<td>1972</td>
<td>3.1833467</td>
<td>0.0000082</td>
<td>Crowe</td>
</tr>
<tr>
<td>1977</td>
<td>3.1833403</td>
<td>0.0000044</td>
<td>Casperson; muonium HFS</td>
</tr>
<tr>
<td>1982</td>
<td>3.1833461</td>
<td>0.0000011</td>
<td>Mariam; muonium HFS</td>
</tr>
<tr>
<td>1982</td>
<td>3.1833441</td>
<td>0.0000017</td>
<td>Klempt</td>
</tr>
<tr>
<td>1999</td>
<td>3.18334513</td>
<td>0.00000039</td>
<td>Liu; muonium HFS</td>
</tr>
</tbody>
</table>

$d\lambda/\lambda = 120\text{ppb} \rightarrow$ with theory $35\text{ppb}$

Would need independent measurement of $m_\mu/m_p$ and $g_p$ to convert to $g_\mu$.

- $\mu_e/\mu_p : 8 \times 10^{-9}$
- $m_e/m_p : 4 \times 10^{-10}$
- $g_e : 7 \times 10^{-13}$
- Proton magnetic moment: $d\mu_p/\mu_p \sim 10^{-8} \rightarrow da_\mu/a_\mu > 10^{-5}$
1958 : First EDM experiment

\[ \frac{d \vec{S}}{dt} = -\vec{\mu} \times \vec{B} - \vec{d} \times (\vec{v} \times \vec{B}) \]

**ELECTRIC DIPOLE MOMENT OF THE MUON**

D. Berley, R.L. Garwin, G. Gidal, and L. M. Lederman

Columbia University, New York, New York

(Received July 18, 1958)

This corresponds to a unit charge multiplied by a distance of \((1.1 \pm 0.9) \times 10^{-15}\) cm.

| \eta | < 0.005

EDM → S along B
CERN
Muons in Flight
applied very roughly to the muon by Garwin et al.\textsuperscript{13}

If muons are made to circle in a magnetic field, the spin turns $(1 + \gamma a)$ times as fast as the momentum vector, with the result that after time $t$ in a magnetic field $B$ the relative angle between spin and momentum is changed by the amount $\theta = aB\omega_0 t$, where $\omega_0$ is the cyclotron frequency for zero-energy muons in unit magnetic field.\textsuperscript{14,15}

We wish to report here the first precision measurement of the muon anomalous moment by this method.\textsuperscript{16}
The Thomas-BMT Equation

\[
\frac{d \mathbf{s}}{dt} = \frac{e}{mc} \mathbf{s} \times \left[ \left( a + \frac{1}{\gamma} \right) \mathbf{B} + \frac{a \gamma}{\gamma + 1} (\mathbf{\beta} \cdot \mathbf{B}) \mathbf{\beta} - \left( \frac{g}{2} - \frac{\gamma}{\gamma + 1} \right) \mathbf{\beta} \times \mathbf{E} \right]
\]

\[
\frac{d \mathbf{\beta}}{dt} = \frac{e}{\gamma mc} \left[ \mathbf{E} + \mathbf{\beta} \times \mathbf{B} - \mathbf{\beta} (\mathbf{\beta} \cdot \mathbf{E}) \right]
\]

\[
\omega_a = \omega_S - \omega_\beta = a \frac{e}{mc} B
\]

Instantly gain a factor \(\sim 1000\) in precision ...
CERN I : Graded dipole field

MEASUREMENT OF THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

CERN, Geneva, Switzerland
(Received January 16, 1961)

\[ a_{\text{exp}} = a_{\text{th}} (0.983 \pm 0.019) = 0.001145 \pm 0.000022, \]

second-order QED
Muon Decay in Flight: Angle-to-Energy

\[ p_L = \gamma (\beta E_{CM}^M + p_{CM}^M \cos \theta_{CM}^M), \quad p_T = p_{CM}^M \sin \theta_{CM}^M \]

Emission angle in CM is correlated to spin direction

E-Energy in LAB is correlated to spin direction
CERN II: The first storage ring

- Proton injection
- Multi-turn storage
- Momentum 1.27 GeV/c
- Magnetic focussing

Third-order QED + almost hadronic correction

\[ a_\mu^+ = (11657.5 \pm 7.1) \times 10^{-7} \]
\[ a_\mu^- = (11662.5 \pm 2.4) \times 10^{-7} \]
The magic momentum

\[ \vec{\omega}_a = -\frac{e}{mc} \left[ a \vec{B} + \left( a - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right] \]

Zero for $\gamma=29.3$, $p_\mu=3.094$ GeV/c

Electric instead of magnetic focussing

Use pure magnetic dipole for precise B measurement
Magnetic Field Uniformity

Measure

\[ \omega_a = \frac{a_\mu}{g_\mu} \mu_\mu B = \frac{a_\mu}{2(a_\mu + 1)} \mu_\mu B \]

\[ \omega_p = \mu_p B \]

\[ a_\mu = \frac{R}{\lambda - R} \]

\[ R = \frac{\omega_a}{\omega_p} \]

\[ \lambda = \frac{\mu_\mu}{\mu_p} \]

If \( B(\text{muons}) = B(\text{protons}) \)

Need homogenous field
Need stable field
Need \( \lambda \)

No need to (precisely) know where the muons are

40 separately stabilized magnets
Inflector to reduce non-uniformity
Proton NMR probes outside vacuum
Remove chambers to probe inside
CERN III: beyond QED

NEW MEASUREMENT OF \((g-2)\) OF THE MUON

J. BAILEY\(^1\), K. BORER\(^2\), F. COMBLEY\(^3\), H. DRUMM\(^4\), C. ECK\(^4\), F.J.M. FARLEY\(^5\), J.H. FIELD, W. FLEGELE, P.M. HATTERSLEY\(^6\), F. KRIENEN, F. LANGE\(^4\), G. PETRUCCI, E. PICASSO, H.I. PIZER, O. RUNOLFSSON, R.W. WILLIAMS\(^7\) and S. WOJCICKI\(^8\)

*CERN Muon Storage Ring Collaboration\(^9\)

Received 4 February 1975

Confirms hadronic contribution
The electric dipole moments were also measured with the results $D^+ = (8.6 \pm 4.5) \times 10^{-19} \, e \cdot cm$ and $D^- = (0.8 \pm 4.3) \times 10^{-19} \, e \cdot cm$. Under the assumption of the CPT theorem these yield a weighted average of $D = (3.7 \pm 3.4) \times 10^{-19} \, e \cdot cm$. 

**EDM up-down asymmetry 90° out of phase w.r.t. g-2**
The electric dipole moments were also measured with the results $D_\mu^+ = (8.6 \pm 4.5) \times 10^{-19} \text{ e} \cdot \text{cm}$ and $D_\mu^- = (0.8 \pm 4.3) \times 10^{-19} \text{ e} \cdot \text{cm}$. Under the assumption of the CPT theorem these yield a weighted average of $D_\mu = (3.7 \pm 3.4) \times 10^{-19} \text{ e} \cdot \text{cm}$. 

EDM up-down asymmetry 90° out of phase w.r.t. g-2
Many improvements
- muon injection
- continuous storage ring
- superconducting magnet
- in-situ magnetometry
- high-resolution calorimeters
- WFDs
- record intensity
- circular aperture
- calibration probe
- …

… and many more (a lot younger!) collaborators
Beam intensity & purity

- Move to most intense proton source: AGS 70TPOT in 12 bunches every 2.7s
- Use ~90m long pion decay line about 50% decay, cut pions before ring
- Directly inject muons using fast kickers ~200ns wide pulse
Homogenous B-field

- C-shaped iron yoke
- \( \rho = 7 \text{m} \) super-conducting coil
- Shielded SC inflector
- Precision iron poles
- Wedges to shim azimuthal variations & quadrupole moment
- Rose shims
- Current shims

same as in \( \lambda \) measurement

- Calibration via spherical water probe
- Plunging probe to transfer calibration
- Fixed probes to track time variation
- NMR trolley to scan storage volume
Magnetic field tracking

Beam profile weighing & time interpolation

φ averaging & (x,y) multipole expansion
Improved electron detection

- Good energy resolution (few %)
- Fast response (few ns)
- Compact (half a shoebox)
- WFD readout (400MHz)
- Gated off during “flash”

Detailed offline analysis possible

~$10^{10}$ events
Double Blind Analysis

\[ R = \frac{\omega_a}{\omega_p} \]
\[ a_\mu = \frac{R}{\lambda - R} \]
\[ \lambda = \frac{\mu_\mu}{\mu_p} \]

- \(\omega_a\) and \(\omega_p\) analyzed independently
  - \(\Rightarrow\) unknown offsets
- \(\omega\)'s analyzed independently by several groups (2–5)
  - \(\Rightarrow\) unknown offsets
- require internal consistency
  - \(\Rightarrow\) \(\omega\)'s equal within (highly correlated) statistics, ...
- require mutual consistency (incl. systematics)
  - \(\Rightarrow\) \(\omega\)'s equal within (highly correlated) statistics, ...
- obtain all systematics (incl. E-field, pitch, ...)
  - \(\Rightarrow\) understand differences, ...
- remove offset, do long-division and publish
  - \(\Rightarrow\) no room for discussion!
Final result

Combined average ($\mu^+$ and $\mu^-$)

$$a_\mu(\text{Expt.}) = 11,659,208.0(5.3)(3.3) \times 10^{-10}(0.54\text{ppm})$$

(statistics limited)
## Storage ring results

<table>
<thead>
<tr>
<th>Year</th>
<th>$a_\mu$</th>
<th>$\sigma a_\mu$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1961</td>
<td>0.001145</td>
<td>0.000022</td>
<td>Charpak</td>
</tr>
<tr>
<td>1965</td>
<td>0.001162</td>
<td>0.000005</td>
<td>Charpak</td>
</tr>
<tr>
<td>1966</td>
<td>0.001165</td>
<td>0.000003</td>
<td>Farley</td>
</tr>
<tr>
<td>1969</td>
<td>0.001060</td>
<td>0.000067</td>
<td>Henry</td>
</tr>
<tr>
<td>1972</td>
<td>0.00116616</td>
<td>0.00000031</td>
<td>Bailey</td>
</tr>
<tr>
<td>1975</td>
<td>0.001165895</td>
<td>0.00000027</td>
<td>Bailey</td>
</tr>
<tr>
<td>1979</td>
<td>0.001165910</td>
<td>0.00000012</td>
<td>Bailey</td>
</tr>
<tr>
<td>1979</td>
<td>0.001165936</td>
<td>0.00000012</td>
<td>Bailey</td>
</tr>
<tr>
<td>1999</td>
<td>0.001165925</td>
<td>0.000000015</td>
<td>Carey</td>
</tr>
<tr>
<td>2000</td>
<td>0.0011659191</td>
<td>0.0000000059</td>
<td>Brown</td>
</tr>
<tr>
<td>2001</td>
<td>0.0011659202</td>
<td>0.000000015</td>
<td>Brown</td>
</tr>
<tr>
<td>2002</td>
<td>0.0011659204</td>
<td>0.000000009</td>
<td>Bennett</td>
</tr>
<tr>
<td>2004</td>
<td>0.0011659214</td>
<td>0.000000009</td>
<td>Bennett</td>
</tr>
</tbody>
</table>

$\mu$ (expt.) - $\mu$ (th) = $(30 \pm 8) \times 10^{-10}$ (3.6$\sigma$)

### Experiment:

$\sigma_a/a = 0.54$ ppm

### Theory:

$\delta_a/a \approx 0.44$ ppm
As at CERN, measure up-going vs down-going electron asymmetry

NEW: reconstruct vertical angle (much better systematic, much less statistics)

\[ |d_\mu| < 1.8 \times 10^{-19} \text{ e}\cdot\text{cm} \]

Bennett et al.,
PRD 80, 052008 (2009)

10^{17} to go for SM
g-2 @ FNAL
Build on a proven formula
Expected situation after experiment:

- **Experimental uncertainty:** $63 \rightarrow 16 \times 10^{-11}$
  - 0.1 ppm statistical $\rightarrow 21 \times$ the E821 events
  - 0.1 ppm systematic overall
    - 0.07 ppm field $\rightarrow 0.17 \rightarrow 0.07$
    - 0.07 ppm $\omega_a$ $\rightarrow 0.21 \rightarrow 0.07$

- **Theory uncertainty:** $51 \rightarrow 30 \times 10^{-11}$

+ improve EDM limit by a factor 100
Why FNAL?

- **Build on a proven formula**
  - E821 → studies to improve at BNL, J-PARC, or FNAL → P989
    - Many studies completed, much documentation
    - Shovel-ready experiment

- **Why Fermilab is uniquely appropriate**
  - Aligned with laboratory direction toward Intensity Frontier
    - For example, synergistic with Mu2e
  - Runs parasitically with Main-Injector neutrinos
    - Efficient use of facility
  - Proton intensity and beam structure ideal for required statistics
    - $1.8 \times 10^{11}$ events in final fits
  - Reduced hadronic-induced background at injection
    - Long decay beamline is key to reducing many systematic errors
  - Increased fill frequency reduces instantaneous rate
    - x4 at FNAL compared to BNL
Using the pbar-accumulator

Using a 900m long decay channel reduces pion content by 20 and gives 6-12 times more stored muons per proton.

<table>
<thead>
<tr>
<th>parameter</th>
<th>BNL</th>
<th>FNAL</th>
<th>gain factor FNAL/BNL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_\pi$ pion/p into channel acceptance</td>
<td>$\approx 2.7E-5$</td>
<td>$\approx 1.1E-5$</td>
<td>0.4</td>
</tr>
<tr>
<td>L decay channel length</td>
<td>88 m</td>
<td>900 m</td>
<td>2</td>
</tr>
<tr>
<td>decay angle in lab system</td>
<td>$3.8 \pm 0.5$ mr</td>
<td>forward</td>
<td>3</td>
</tr>
<tr>
<td>$\delta p_\pi/p_\pi$ pion momentum band</td>
<td>$\pm 0.5%$</td>
<td>$\pm 2%$</td>
<td>1.33</td>
</tr>
<tr>
<td>FODO lattice spacing</td>
<td>6.2 m</td>
<td>3.25 m</td>
<td>1.8</td>
</tr>
<tr>
<td>inflector</td>
<td>closed end</td>
<td>open end</td>
<td>2</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td></td>
<td>11.5</td>
</tr>
</tbody>
</table>
Timeline

- **2009**
  - PAC presentation / Stage-1 approval
  - Some R&D funds made available

- **Year 0**
  - Planning / designs / tests

- **Year 1**
  - Build new polarized target complex
  - Order, electronics tests and pre-construction
  - Re-machine fixed probe locations on vacuum chambers

- **Year 2**
  - Install and assemble ring at FNAL
  - Complete modifications of beamline
  - Special rate tests of pion beam
  - Detector, electronics, commission

- **Year 3**
  - Detector, electronics, commission
  - Sensors, integrate counting room, DAQ
  - Physics commissioning
  - Start real data taking

**Future:** $\Delta a_\mu(\text{Expt} - \text{Thy}) = xx \pm 34 \times 10^{-11}$

DOE review scheduled for Aug. 2010

Detailed plan of approach
The big move: BNL → FNAL

- Transport coils to and from barge via Sikorsky aircrane
- Ship through St Lawrence → Great Lakes → Calumet SAG
- Subsystems can be transported overland, but probably more cost effective to ship steel on barge as well.

Courtesy Chris Polly
Frozen Spin
EDM @ FNAL, PSI, J-PARC
Freezing the spin

\[ \vec{\omega} = \frac{e}{m} \left[ a \hat{B} + \left( a - \frac{1}{\gamma^2 - 1} \right) \vec{v} \times \vec{E} + \frac{\eta}{2} \left( \vec{E} + \vec{v} \times \vec{B} \right) \right] \]

E=0, \ B=B_y

(1) \( \omega = \sqrt{a^2 + (\eta \beta)^2} / 4 \ B \)

(2) \( \hat{\omega} \times \hat{B} = \eta \beta / 2 a \)

E_r \approx aBc\beta \gamma^2,

B=B_y

(1) \( \hat{\omega} \times \hat{B} = 1 \)
Ideas

A. Silenko et al. (2003)
J-PARC Letter of Intent:
“Search for the Permanent Muon Electric Dipole Moment at the $10^{-24}$ e\cdot cm Level.”

Design:
- $p = 500 \text{MeV/c}$
- $B = 0.25 \text{T}$
- $E = 2 \text{MV/m}$
- $R = 7 \text{m}$

Estimated Sensitivity around $10^{-24}$ e\cdot cm
five orders below current limit

Post-(g-2) @ FNAL
First-guess design
- $p = 600 \text{MeV/c}$
- $B = 0.45 \text{T}$
- $E = 5 \text{MV/m}$
- $R = 4.2 \text{m}$

Estimated Sensitivity around $10^{-24}$ e\cdot cm
five orders below current limit

Idea for PSI
- $p = 125 \text{MeV/c}$
- $N = 2 \cdot 10^5 \text{/s}$
- $P = 92\%$
- $B = 1 \text{ \text{T}}$
- $E = 0.64 \text{MV/m}$
- $R = 42 \text{cm}$
- Reach: $5 \times 10^{-23}$ e\cdot cm / y

3-4 orders below current limit

Table top experiment!

See talks by Semertzidis & Ströher (for p & d)

Finale
Conclusion

Over half a century of experience performing measurements of the dipole moments of the muon have taught us that

- performing a highly precise experiment requires extreme care & many clever ideas
- the muon is a lepton
- the muon behaves like a heavy electron
- QED, QCD & QFD corrections are “correct”
- there might be a hint for new physics

And there is (much) more to come ...
Many thanks to ...