Theory of the Muon $g-2$

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$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\vec{s} \qquad \text{and} \qquad \underbrace{g_\mu = 2}_{\text{Dirac}}(1+a_\mu)$$

$$a_\mu = \frac{1}{2}(g_\mu - 2) \quad : \quad \text{anomalous magnetic moment}$$

World average experimental value:
(dominated by the BNL–experiment)

$$a_\mu(\text{exp.}) = 11\ 659\ 202.3(15.1) \times 10^{-10}\ [1.3\text{ppm}]\ (\text{Sep.'01})$$

$$a_\mu(\text{exp.}) = 11\ 659\ 203 \qquad (8) \times 10^{-10}\ [0.7\text{ppm}]\ (\text{Sep.'02})$$

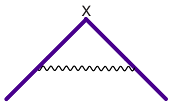
Standard Model "prediction" (September 2001) was:

$$a_\mu^{\text{SM}} = (11\ 659\ 159.7 \pm 6.7) \times 10^{-10}\,.$$

- The 2001 2.6σ "discrepancy" $\Rightarrow$ *avalanche* of theoretical papers.

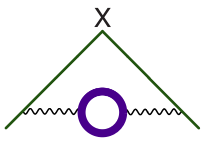
- Today I shall review the present status on the Standard Model prediction.

Some Theoretical Comments



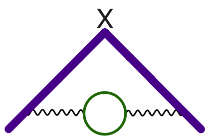
$\Rightarrow \quad a_\mu = a_e = \frac{1}{2}\frac{\alpha}{\pi}$ Schwinger '48

- Loops with *different masses* $\Rightarrow a_\mu \neq a_e$
  - Internal *LARGE* masses decouple:



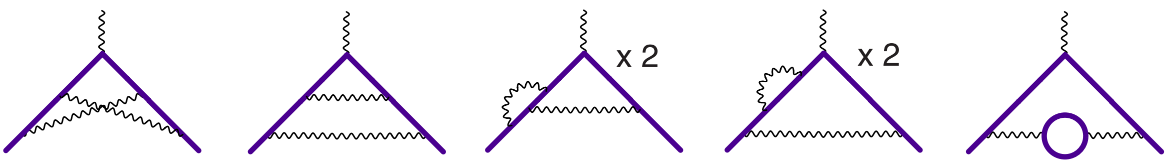
$$\Rightarrow \left[\left(\frac{1}{3}\right)\left(\frac{1}{15}\right)\left(\frac{m_\mu}{m_\tau}\right)^2 + \mathcal{O}\left(\frac{m_\mu^4}{m_\tau^4}\log\frac{m_\tau}{m_\mu}\right)\right]\left(\frac{\alpha}{\pi}\right)^2$$

  - Internal *SMALL* masses give rise to log's of mass ratios:



$$\Rightarrow [\underbrace{\left(\frac{2}{3}\right)}_{\beta_1}\left(\frac{1}{2}\right)\log\frac{m_\mu}{m_e} - \frac{25}{36} + \mathcal{O}\left(\frac{m_e}{m_\mu}\right)]\left(\frac{\alpha}{\pi}\right)^2$$

- Two loops: *7 Feynman diagrams (with common fermion lines)*



$$a_l^{(4)} = \left\{\frac{197}{144} + \frac{1}{12}\pi^2 - \frac{1}{2}\pi^2\ln 2 + \frac{3}{4}\zeta(3)\right\}\left(\frac{\alpha}{\pi}\right)^2$$

Peterman '57
Sommerfield '57

- Three loops: *72 Feynman diagrams (which I can show you)* Laporta– Remiddi '96

- Four loops: *891 Feynman diagrams (which I wan't show you)* Kinoshita (in progress)

The Muon Anomaly

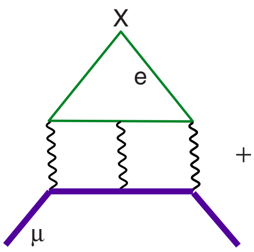
$$a_\mu = [a_e]_{\rm QED} + \underbrace{a_\mu(e,\tau) + a_\mu(\rm hadrons)}_{SU(3)\times SU(2)\times U(1)}$$

- Vacuum Polarization from electron loops
  - Enhanced by QED short–distance logarithms
  - Obey Renormalization Group Equation $[\alpha \Rightarrow \alpha(m_\mu)]$

$$\left(m_e \frac{\partial}{\partial m_e} + \beta(\alpha)\alpha\frac{\partial}{\partial\alpha}\right) a_\mu^{(\infty)}\left(\frac{m_\mu}{m_e},\alpha\right) = 0$$

Lautrup–de Rafael '74

- Light–by–Light Scattering from electron loops



  - Enhanced by QED infrared logarithms

Kinoshita *et al* '69
Laporta–Remiddi '93

$$a_\mu^{(3)}|_{\rm l.byl.} = \left[\frac{2}{3}\pi^2 \ln\frac{m_\mu}{m_e} + \cdots\right]\left(\frac{\alpha}{\pi}\right)^3 = 20.947\ldots\left(\frac{\alpha}{\pi}\right)^3$$

- Vacuum Polarization and Light–by–Light Scattering
  from tau loops, suppressed by Mass Decoupling (but explicitly known)

$$a_\mu(\rm QED) = (11\ 658\ 470.57 \pm 0.29) \times 10^{-10}$$

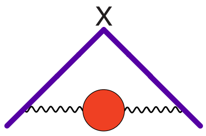
Recall that at present:

$$a_\mu(\rm Exp.) = (11\ 659\ 203 \quad \pm \quad 8) \times 10^{-10}$$

Is this discrepancy due to the SM Hadronic–EW Interactions ?

Hadronic Vacuum Polarization and $g_\mu - 2$

- All calculations are based on the spectral representation



$$a_\mu^{(\text{h. v.p.})} = \frac{\alpha}{\pi}\int_0^\infty \frac{dt}{t}\frac{1}{\pi}\text{Im}\Pi(t)\ \int_0^1 \frac{x^2(1-x)}{x^2+\frac{t}{m_\mu^2}(1-x)}dx$$

$$\sigma(t)_{e^+e^-\to\text{hadrons}} = \frac{4\pi^2\alpha}{t}\frac{1}{\pi}\text{Im}\Pi(t)$$

$$\left(\frac{1}{q^2}\right) \Rightarrow \int_0^\infty \frac{dt}{t}\left(\frac{1}{q^2-t}\right)\frac{1}{\pi}\text{Im}\Pi(t)$$

- Adler Function (Relevant QCD Function, $Q^2 = -q^2 > 0$)

$$\mathcal{A}(Q^2) = -Q^2\frac{\partial\Pi(Q^2)}{\partial Q^2} = \int_0^\infty dt\frac{Q^2}{(t+Q^2)^2}\frac{1}{\pi}\text{Im}\Pi(t)$$

$$a_\mu^{(\text{h. v.p.})} = \frac{\alpha}{\pi}\int_0^1 dx\frac{1}{x}(1-x)(1-x/2)\mathcal{A}\left(\frac{x^2}{1-x}m_\mu^2\right)$$

- Large N_c QCD *Minimal Hadronic Ansatz* Approximation

$$\mathcal{A}(Q^2) = \left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right)e^2\left\{2f_V^2M_V^2\frac{Q^2}{(Q^2+M_V^2)^2} + \frac{N_c}{16\pi^2}\frac{4}{3}\frac{Q^2}{Q^2+s_0}(1+\cdots)\right\}$$

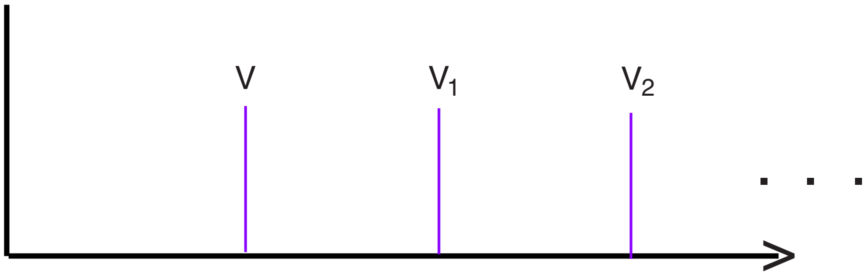
  - No $1/Q^2$ term in the OPE $\Rightarrow$

$$2f_V^2M_V^2 = \frac{N_c}{16\pi^2}\frac{4}{3}s_0\left(1+\frac{3}{8}\frac{\alpha_s(s_0)}{\pi}+\cdots\right)$$

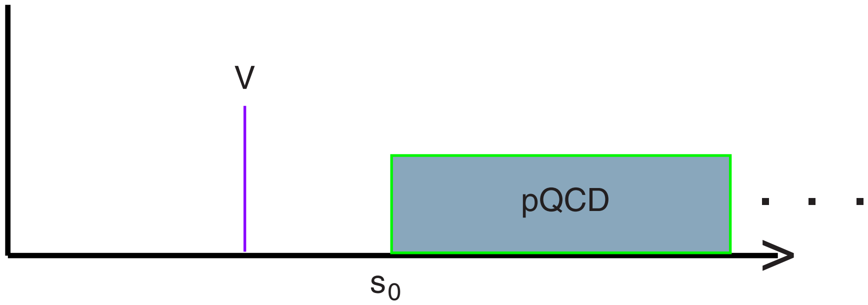
  - Chiral loops (two–pion states) subleading in $1/N_c$

$$a_\mu^{(\text{h. v.p.})} \simeq (570 \pm \underbrace{170}_{30\%\ \text{sys. error}}) \times 10^{-10}$$

The Large–N_c World



Minimal Hadronic Approximation

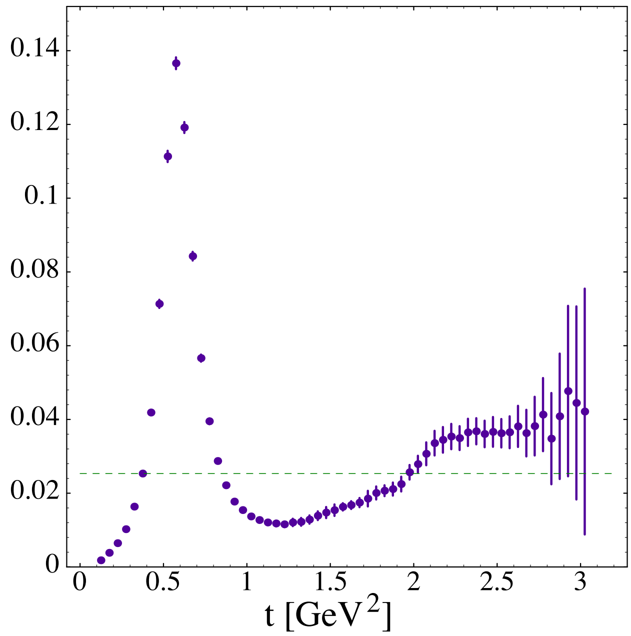


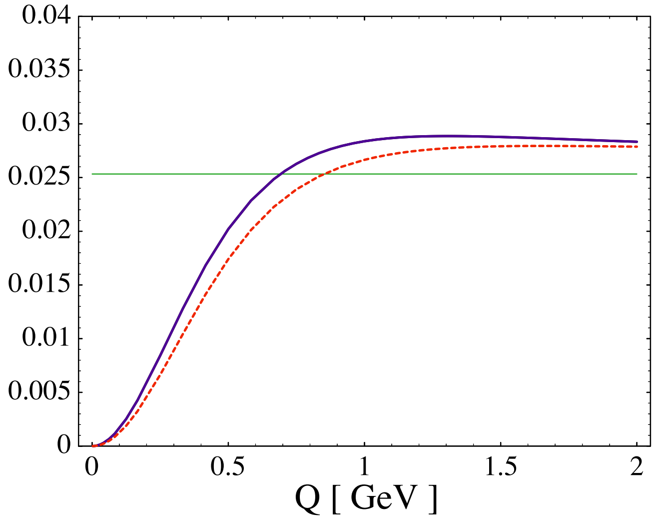
$$\mathcal{A}(Q^2)=\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right)e^2\left\{2f_V^2 M_V^2\frac{Q^2}{(Q^2+M_V^2)^2}+\frac{N_c}{16\pi^2}\frac{4}{3}\frac{Q^2}{Q^2+s_0}(1+\cdots)\right\}$$

The Vector Spectral Function (ALEPH–Data)

versus

The Adler Function

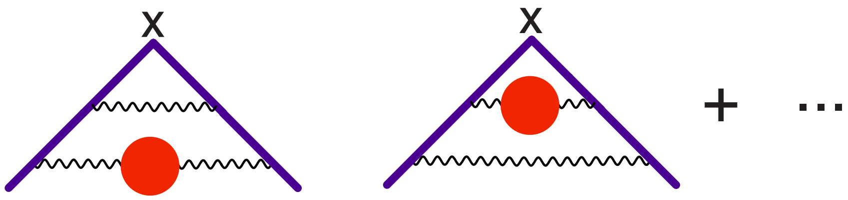




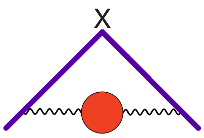
Higher Order Hadronic Vacuum Polarization

Calmet, Narison, Perrottet, de Rafael '76; Krause '97
Francis FARLEY's question Friot, Greynat (in progress)

- Hadronic Vacuum Polarization insertion in each photon propagator of the two–loop diagrams (7×2 diagrams)



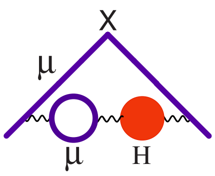
Recall the lowest order HVP calculation



$$\Rightarrow a_\mu^{(2)} = \frac{\alpha}{\pi}\int_{4M_\pi^2}^\infty \frac{dt}{t}\frac{1}{\pi}\mathrm{Im}\Pi(t)\underbrace{\int_0^1 \frac{x^2(1-x)}{x^2+\frac{t}{m_\mu{}^2}(1-x)}dx}_{\sim\frac{1}{3}\frac{m_\mu^2}{t}}$$

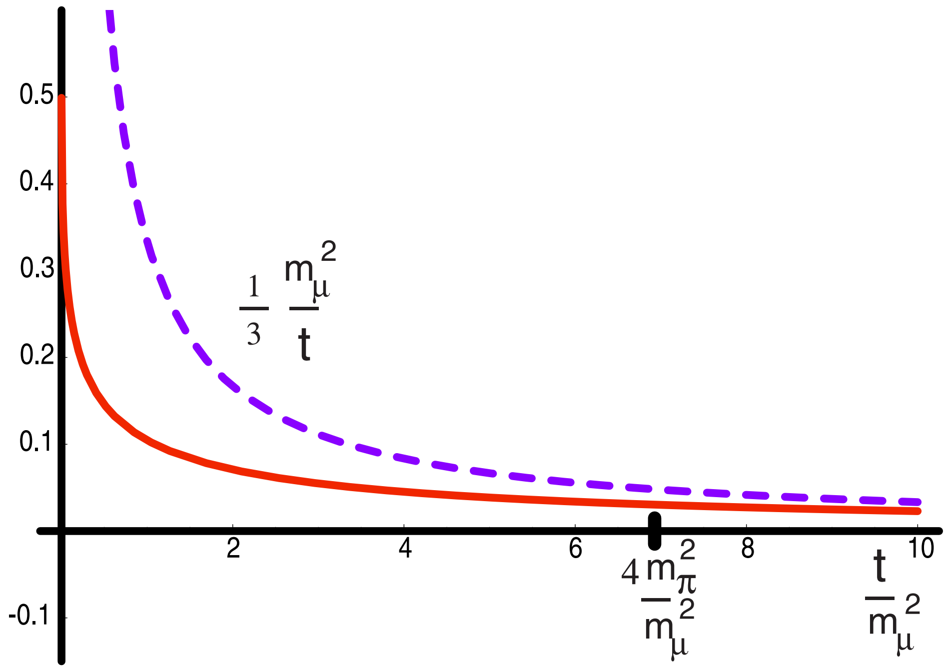
This is now replaced by

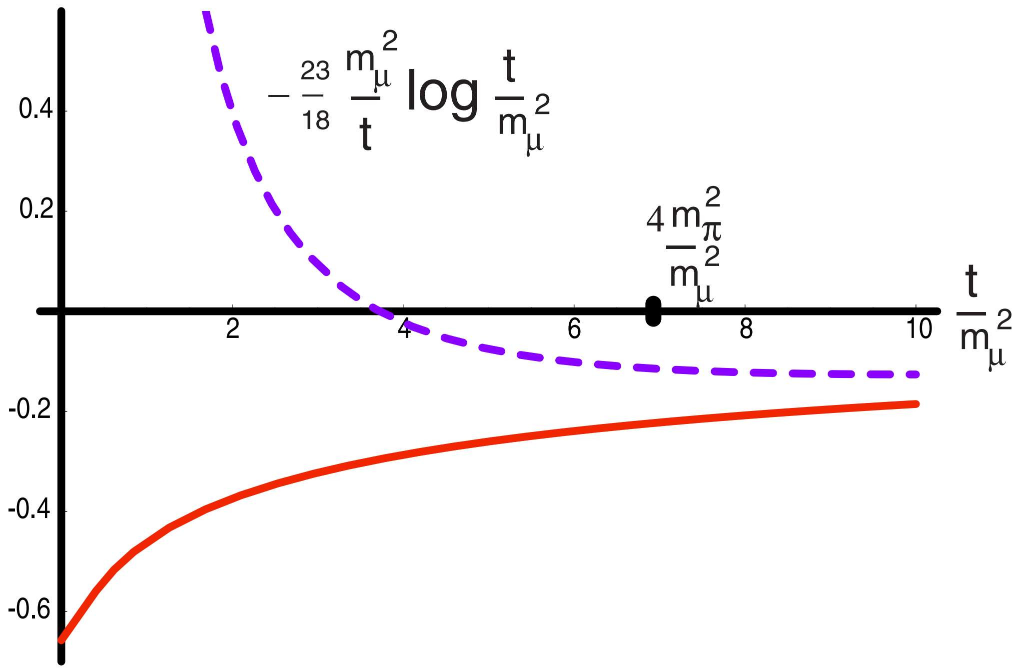
$$\left(\frac{\alpha}{\pi}\right)^2\int_{4M_\pi^2}^\infty \frac{dt}{t}\frac{1}{\pi}\mathrm{Im}\Pi(t)\left\{\underbrace{-\frac{23}{18}\frac{m_\mu^2}{t}\log\left(\frac{t}{m_\mu^2}\right)}_{\text{negative}}+\mathcal{O}\left(\frac{m_\mu^2}{t}\right)\right\}$$

Notice that 

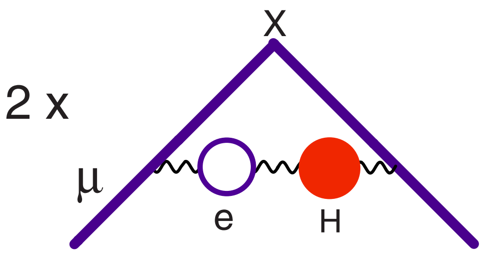
(positive) only contributes to $\mathcal{O}\left(\frac{m_\mu^2}{t}\right)$

The QED (Massive Photon) Convolution Functions





Hadronic Vacuum Polarization insertions in the $\log\frac{m_\mu}{m_e}$ enhanced VP diagram



$$\frac{\alpha}{\pi}\int_{4M_\pi^2}^{\infty}\frac{dt}{t}\frac{1}{\pi}\mathrm{Im}\Pi(t)\ \underbrace{\int_0^1 \frac{x^2(1-x)}{x^2+\frac{t}{m_\mu{}^2}(1-x)}dx}_{\sim\frac{1}{3}\frac{m_\mu^2}{t}}\ \underbrace{(-2)\times\Pi\left(\frac{x^2}{1-x}\frac{m_\mu^2}{m_e^2}\right)}_{\sim(-2)\frac{\alpha}{\pi}\left(-\frac{1}{3}\right)\log\frac{m_\mu^2}{m_e^2}}$$

Therefore, the total is

$$a_\mu^{(4)}\sim\left(\frac{\alpha}{\pi}\right)^2\int_{4M_\pi^2}^{\infty}\frac{dt}{t}\frac{m_\mu^2}{t}\frac{1}{\pi}\mathrm{Im}\Pi(t)\left[-\frac{23}{18}\log\frac{t}{m_\mu^2}+\frac{2}{9}\log\frac{m_\mu^2}{m_e^2}\right]$$

and the negative contribution wins (because $<t>\sim M_\rho^2$)

$$a_\mu^{(4)}=-(10.1\pm0.6)\times10^{-10}$$ Krause 97

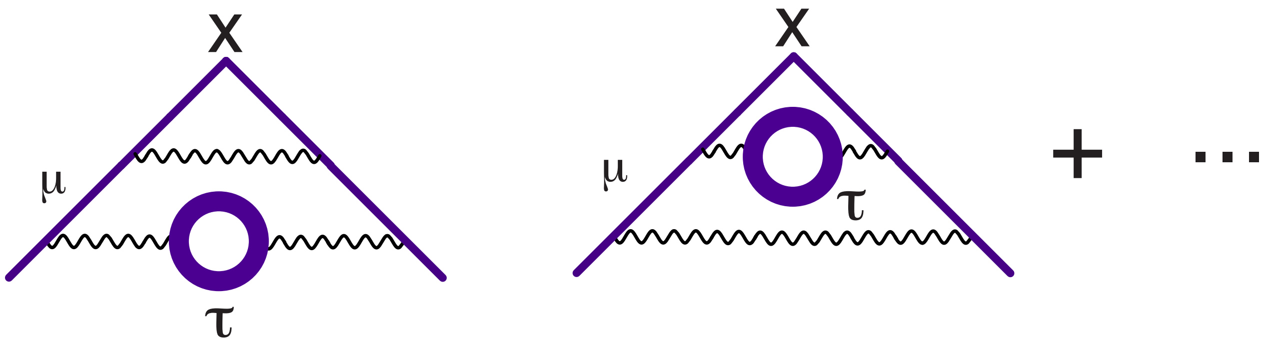
* * *

COMMENT on the Universality of the $-23/18$ factor:

Anomalous dimensions of effective operators when HEAVY degrees of freedom have been integrated out

Higher Order τ–Vacuum Polarization insertions in the muon $g-2$

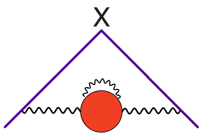
(Same as μ–VP insertions in electron T. Kinoshita's talk)



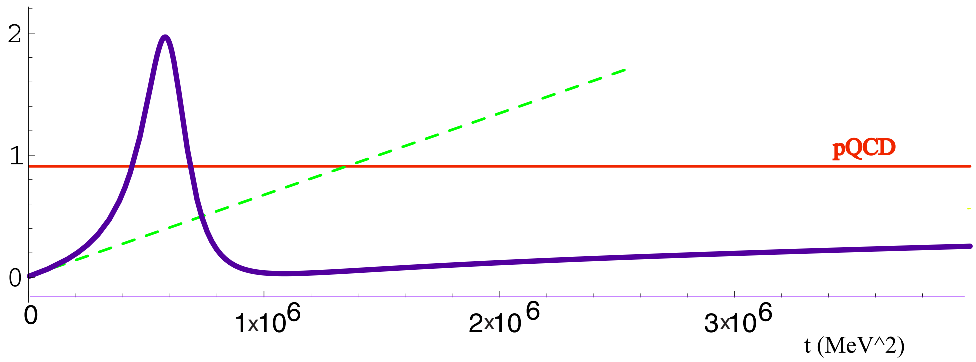
Leading Contribution:

$$a_\mu^{(\tau)} = \left(\frac{\alpha}{\pi}\right)^3 \underbrace{\left(-\frac{23}{18}\right)}_{\text{anom. dim.}} \times \underbrace{\left(\frac{1}{15}\right)}_{\text{slope VP}} \frac{m_\mu^2}{m_\tau^2} \log \frac{m_\tau^2}{m_\mu^2}$$

Hadronic Vacuum Polarization
with EM Self–Energy



Calculation (Perrottet *et al* in progress) of the corresponding Hadronic Spectral Function (in 10^{-6} units) :



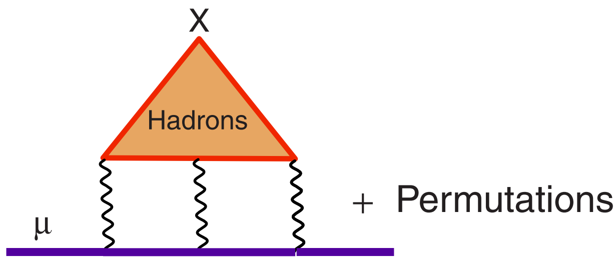
Large–N_c inspired calculation with χPT constraints and short–distance pQCD incorporated ($\mathcal{L}_{P\gamma\gamma}$, $\mathcal{L}_{V\gamma}$, $\mathcal{L}_{VP\gamma}$ and $\mathcal{L}_{VVP}$ so far...)

$$a_\mu|_{\text{preliminary}} = (39.5 \pm 0.4 \pm \text{s.error}) \times 10^{-11}$$

$$\text{From } \rho \text{ only} = (12.1 \pm 0.1 \pm \text{s.error}) \times 10^{-11}$$

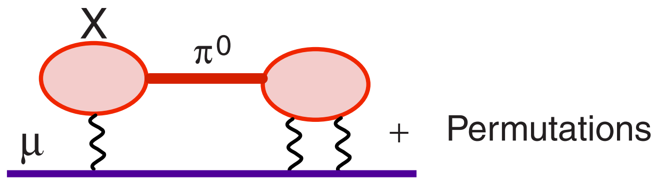
Authors	Contribution to $a_\mu \times 10^{11}$
Achasov-Kiselev	54.7 ± 1.5
de Trocòniz-Ynduràin	43 ± 4
Davier-Eidelman-Hocker-Zhang	$9.3 \pm 1.5 \pm 0.1$

Hadronic L–by–L Scattering



- All Estimates (so far) are model dependent
- Progress in identifying dominant regions of virtual momenta
- Most of the recent estimates use models *compatible* with low–energy χPT behaviour and Large–N_c counting rules (de R '94)
  - ENJL–model Bijnens, Pallante, Prades '96
  - Vector Gauge Model Hayakawa – Kinoshita '98
  - MHA to Large–N_c QCD Knecht–Nyffeler '01

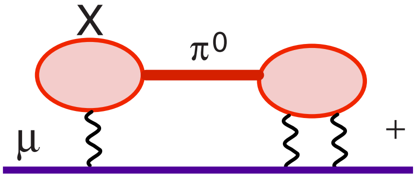
- In these calculations, the dominant contribution comes from twice the anomalous VVP vertex



$$a_\mu^{(\pi^0 \text{ l by l})} = +(5.8 \pm 1.0) \times 10^{-10}$$ Knecht–Nyffeler '01

$$a_\mu^{(\text{H l by l})} = (+8 \pm 4.0) \times 10^{-10}$$

The Knecht–Nyffeler Calculation



$$a_{\mu}^{(\pi^0 \text{ l. by l.})} = \int_0^\infty dQ_1^2 \int_0^\infty dQ_2^2 \;\; \mathcal{W}(Q_1^2, Q_2^2)\mathcal{H}(Q_1^2, Q_2^2)$$

$\mathcal{H}(Q_1^2, Q_2^2)$ is a convolution of two $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$ form factors.

In Large–N_c QCD:

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)\Big|_{N_C\to\infty} = \sum_{ij} \frac{c_{ij}(q_1^2, q_2^2)}{(q_1^2 - M_i^2)(q_2^2 - M_j^2)},$$

with constraints on $c_{ij}(q_1^2, q_2^2)$ from Long–Distance and Short–Distance QCD

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0) = -\frac{N_C}{12\pi^2 F_0},$$

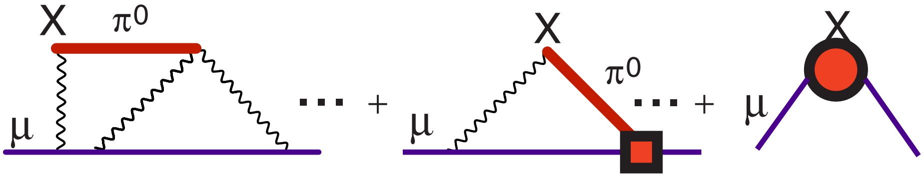
$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}[\lambda^2 q^2, (p-\lambda q)^2] = \frac{2}{3}\frac{F_0}{q^2}\left\{\frac{1}{\lambda^2} + \frac{1}{\lambda^3}\frac{q\cdot p}{q^2}\right\}$$

plus other *phenomenological* constraints

$$\boxed{a_{\mu}^{(\pi^0 \text{ l. by l.})} = +(5.8 \pm 1.0) \times 10^{-10}}$$

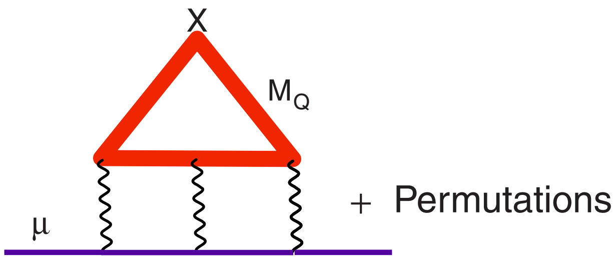
Effective Field Theory Approach

Knecht–Nyffeler–Perrottet–de Rafael '01



$$a_\mu^{(\pi^0)} = \left(\frac{\alpha}{\pi}\right)^3 \left\{ \underbrace{\frac{N_c^2}{48\pi^2}\frac{m_\mu^2}{f_\pi^2}} \log^2\left(\frac{\mu}{m}\right) + \mathcal{O}\left[\log\left(\frac{\mu}{m}\right)\right] + \kappa(\mu) \right\}$$

- Whatever UV– μ, the coefficient of $\log^2 \mu$ is an exact QCD result
- The Knecht–Nyffeler Calculation reproduces this leading behaviour when $M_\rho^2 \to \infty$
- The CQM, is NOT an effective theory of QCD !!!



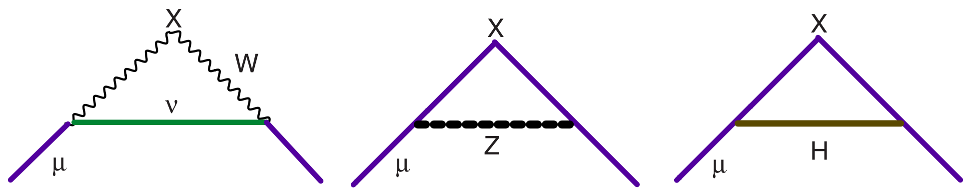
$$a_\mu^{(\mathrm{CQM})} = \left(\frac{\alpha}{\pi}\right)^3 N_c \frac{2}{9} \left\{ \underbrace{\left[\frac{3}{2}\zeta(3) - \frac{19}{16}\right]}_{0.616} \left(\frac{m_\mu}{M_Q}\right)^2 + \mathcal{O}\left[\left(\frac{m_\mu}{M_Q}\right)^4 \log^2\left(\frac{M_Q}{m_\mu}\right)\right] \right\}$$

Laporta–Remiddi '93

Weak Interactions

- One Loop

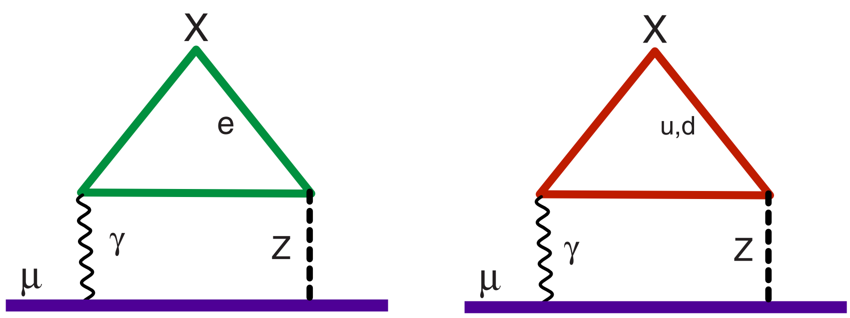
  Bardeen–Gastmans–Lautrup 72, Altarelli–Cabbibo–Maiani 72, Jackiw–Weinberg 72, Bars–Yoshimura 72, Fujikawa–Lee–Sanda 72.



$+38.9 \times 10^{-10}$ $\quad -19.4 \times 10^{-10}$ $\quad \leq 3.3 \times 10^{-14}$

- Two Loops
  - Possible large terms of $\mathcal{O}\left[\left(\frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\right) \times \frac{\alpha}{\pi} \log \frac{M^2}{m_l^2}\right]$
  - Separation of LEPTONS and QUARKS no longer possible

    Peris–Perrottet–de Rafael '95, Czarnecki–Krause–Marciano '95, '96



$$a_\mu^{EW} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2-\left(\frac{\alpha}{\pi}\right)\left(159\,{}^{+4}_{-8}\right)\right]$$

Knecht–Peris–Perrottet–de Rafael 02

Czarnecki-Marciano-Vainshtein 03

$$= \left(15.2\,{}^{+0.2}_{-0.1}\right) \times 10^{-10}$$

Theoretical Comments on the $\gamma\gamma Z$ Triangle

Knecht–Peris–Perrottet–de Rafael 02
Czarnecki-Marciano-Vainshtein 03, Vainshtein 03

The "Battle Horse" is (for k small)

$$W_{\mu\nu\rho}(q,k) = \int d^4x\, e^{iq\cdot x} \int d^4y\, e^{i(k-q)\cdot y} \langle \Omega\, | T\{V_\mu(x) A_\nu(y) V_\rho(0)\} | \Omega \rangle$$

$$= \frac{iN_c}{18\pi^2} \Big\{ q_\nu\, \epsilon_{\mu\rho\alpha\sigma}\, q^\alpha k^\sigma\; w_L(Q^2) +$$

$$\left[q_\mu\, \epsilon_{\nu\rho\alpha\sigma}\, q^\alpha k^\sigma - q^2 \epsilon_{\mu\nu\rho\sigma} k^\sigma - q_\nu \epsilon_{\mu\rho\alpha\sigma} q^\alpha k^\sigma \right]\; w_T(Q^2) \Big\} + \mathcal{O}(k^2)$$

The $g-2$ calculation requires the integral

$$\frac{1}{M_Z^2} \int_0^\infty dQ^2 \frac{M_Z^2}{Q^2 + M_Z^2} w_T(Q^2)\,,$$

- THEOREM (Vainshtein 03) In pQCD:
  $w_T(Q^2) = \frac{1}{2} w_L(Q^2) = \frac{1}{Q^2}$ (with NO α_s–corrections)
- HOWEVER In QCD
  $\lim_{Q^2\to 0} w_T(Q^2) \to \frac{1}{M_H^2}$ (there is no pion pole)
- QUESTION Does the pQCD $\frac{2}{Q^2}$ behaviour survive at large Q^2?

  KPPdeR $\Rightarrow \lim_{Q^2\to\infty} w_T(Q^2) = \frac{<d=4>}{Q^6}$

  CMV $\Rightarrow \lim_{Q^2\to\infty} w_T(Q^2) = \frac{1}{Q^2} + \frac{<d=4>}{Q^6}$
- PUZZLE ? The CMV pQCD choice implies an exact SD sum rule

$$\int_0^\infty dt \,\mathrm{Im}\, w_T(Q^2) = \begin{cases} 1 & \text{only to leading } N_c \\ 0 & \text{to any subleading order} \end{cases}$$

Electroweak Results from $\gamma\gamma Z$

Knecht–Peris–Perrottet–de Rafael 02
Czarnecki-Marciano-Vainshtein 03

- Third Generation:

$$\frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times\left[-3\log\frac{M_Z^2}{m_\tau^2}-\log\frac{M_Z^2}{m_b^2}-\frac{8}{3}\log\frac{m_t^2}{M_Z^2}+\frac{8}{3}+\mathcal{O}\left(\frac{M_Z^2}{m_t^2}\log\frac{m_t^2}{M_Z^2}\right)\right]$$

$$=\frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times(-30.6)\,.$$

- Second and First Generation ?

$$\frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times\left\{-3\log\frac{M_Z^2}{m_\mu^2}-\frac{5}{2}\right.$$

$$-3\log\frac{M_Z^2}{m_\mu^2}+4\log\frac{M_Z^2}{m_c^2}-\frac{59}{6}+\frac{8}{9}\pi^2$$

$$\left.+\left[\frac{4}{3}\log\frac{M_Z^2}{m_\mu^2}+\frac{2}{3}+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)\right]+4.57\pm1.80+0.04\pm0.02\right\}$$

$$=\frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times\begin{pmatrix}-28.5\pm1.8\\-24.6\end{pmatrix}\,,$$

Full Electroweak Contribution to Two Loops

♣ $$a_\mu^{EW}=\frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2-\left(\frac{\alpha}{\pi}\right)\left(159\,^{+4}_{-8}\right)\right]$$

$$=\left(15.2\,^{+0.2}_{-0.1}\right)\times10^{-10}\,,$$

- Includes error from Higgs contribution Czarnecki–Krause–Marciano '95

Summary of SM Contributions

- Leptonic QED contributions

$$a_{\rm QED}(\mu) = \ 11\ 658\ 470.57 \pm 0.29 \times 10^{-10}$$

- Hadronic Contributions
  - Vacuum Polarization (Hagiwara *et al* Sept. 02)

$$a^{(\rm VP)}_{\rm hadronic} = [\underbrace{683.1 \pm 7.9}_{\rm LO} - \underbrace{10.0 \pm 0.6}_{\rm HO}]) \times 10^{-10}$$

(Wait however for Michel Davier's talk...)

  - Light–by–Light (After Knecht–Nyffeler's hadronic light-by-light calculation)

$$a^{(\rm light\ by\ light)}_{\rm hadronic} = \underbrace{(8 \pm 4)}_{\rm could\ be\ improved} \times 10^{-10}$$

- Electroweak Contributions (after calculations reported here)

$$a_{\rm EW} = \left(15.2 \begin{array}{c} +0.2 \\ -0.1 \end{array}\right) \times 10^{-10}$$

- Total Standard Model Contribution
  (with above quoted HVP...)

$$a_\mu^{\rm SM} = (11\ 659\ 167 \quad \pm 9) \times 10^{-10}$$
$$a_\mu^{\rm exp} = (11\ 659\ 203 \quad \pm 8) \times 10^{-10}$$

$$\boxed{a_\mu^{\rm exp} - a_\mu^{\rm SM} = (36 \pm 12) \times 10^{-10} \quad 3\sigma}$$

Summary of SM Contributions

- Leptonic QED contributions

$$a_{\text{QED}}(\mu) = 11\,658\,470.57 \pm 0.29 \times 10^{-10}$$

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  - Vacuum Polarization (Hagiwara *et al* Sept. 02)

$$a^{(\text{VP})}_{\text{hadronic}} = [\underbrace{683.1 \pm 7.9}_{\text{LO}} - \underbrace{10.0 \pm 0.6}_{\text{HO}}]) \times 10^{-10}$$

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$$a^{(\text{light by light})}_{\text{hadronic}} = \underbrace{(8 \pm 4)}_{\text{could be improved}} \times 10^{-10}$$

- Electroweak Contributions (after calculations reported here)

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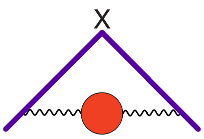
Is there more EVIDENCE for NEW PHYSICS ?
than for ARMS of MASSIVE DESTRUCTION in IRAK ???

Hadronic Vacuum Polarization Results

- All calculations are based on the spectral representation



$$a_\mu^{(\text{h. v.p.})} = \frac{\alpha}{\pi}\int_0^\infty \frac{dt}{t}\frac{1}{\pi}\text{Im}\Pi(t)\ \int_0^1 \frac{x^2(1-x)}{x^2+\frac{t}{m_\mu{}^2}(1-x)}dx$$

$$\sigma(t)_{e^+e^-\to\text{hadrons}} = \frac{4\pi^2\alpha}{t}\frac{1}{\pi}\text{Im}\Pi(t)$$

Compilation from Recent Estimates (with comments):

Authors	**Contribution to** $a_\mu \times 10^{10}$
Davier–Höcker	692.4 ± 6.2
Jegerlehner	697.40 ± 10.45
Narison (last v.)	703.6 ± 7.6
de Trocóniz–Ynduráin (v5)	695.2 ± 6.4
Davier *et al* CMD-2	$684.7 \pm 6.0_{\text{exp}} \pm 3.6_{\text{rad}}$
Davier *et al* ALEPH-CLEO	$701.9 \pm 4.7_{\text{exp}} \pm 1.2_{\text{rad}} \pm 3.8_{SU(2)}$
Hagiwara *et al*	$683.1 \pm 5.9_{\text{exp}} \pm 2.0_{\text{rad}}$

- Problems with *possible* double counting
- Higher Order Hadronic Vacuum Polarization
  Calmet, Narison, Perrottet, de Rafael '77; Krause '97

$$a_\mu^{(\text{h.o.-h. v.p.})} = -10.0\ (0.6) \times 10^{-10}$$

- Question about Hadronic EM self–energy estimates: