Overview of Muon $(g - 2)$ and EDM Experiments

What we have learned from BNL E821

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BNL AGS E821:

A New Precision Measurement of the Muon $(g - 2)$ Value at the level of 0.35 ppm

Boston University, Brookhaven National Laboratory, Budker Institute of Nuclear Physics - Novosibirsk, Cornell University, KEK, KVI and Rijksuniversiteit - Groningen, University of Heidelberg, University of Illinois, University of Minnesota, Tokyo Institute of Technology, Yale University
Outline of the Talk

- Brief Introduction to \((g - 2)\)
- Overview of the Experimental Technique
- Results from E821
- The muon EDM
- Implications for future experiments at JHF
- Summary and Conclusions
Muon: \((2^{nd} \text{ generation lepton})\)

\[
\begin{align*}
m_\mu c^2 &= 105.658\,389(34) \text{ MeV} \\
\tau_\mu &= 2.197\,03(4) \mu\text{s}
\end{align*}
\]

Source: \(\pi^- \rightarrow \mu^- \bar{\nu}_\mu\) \text{ Weak Decay}

Parity Violating Decay \(\Rightarrow\) Polarized Muons

Weak Decay: \(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu\)

The long lifetime facilitates precision measurement.
Magnetic Moments, $g$-Factors, etc.

\[ \vec{\mu}_s = g_s \left( \frac{e}{2m} \right) \vec{s} \]

- $\vec{\mu}$ - magnetic moment; $g$ - gyromagnetic ratio
- $\vec{s}$ is the spin.

- Dirac Equation Predicts $g \equiv 2$
- In nature radiative corrections make $g \neq 2$.

\[ g = 2 + \frac{\alpha}{\pi} + \ldots \]
Magnetic Moments: Definitions and Values

\[ \mu = (1 + a) \frac{e\hbar}{2m} \]

where \( a = \left( \frac{g - 2}{2} \right) \)

\[ \mu_e = 1.001\ 159\ 652\ 193 \ \frac{e\hbar}{2m_e}; \]

For comparison:

\[ \mu_\mu = 1.001\ 165\ 923 \ \frac{e\hbar}{2m_\mu}; \]

\[ \mu_p = 2.792\ 847\ 39 \ \frac{e\hbar}{2m_p} \]

\[ g_p = 5.5857 \cdots \neq 2 \]
Theoretical Value for \((g - 2)\)

- **Electron**: To the level of the experimental error, \(\pm 4\) ppb

\[
a_e(\text{Standard Model}) = a_e(\text{QED with } \gamma, \ e)
\]

Contribution of virtual \(\mu, \tau, \text{ etc.}\) is \(\leq 4\) ppb.
**Theoretical Value for \((g - 2)\)**

- **Electron**: To the level of the experimental error, ±4 ppb

\[ a_e(\text{Standard Model}) = a_e(\text{QED with } \gamma, e) \]

Contribution of virtual \(\mu, \tau, \text{ etc.}\) is \(\leq 4\) ppb.

- **Muon**: The Relative Contribution of heavier things:

\[ \sim \left(\frac{m_\mu}{m_e}\right)^2 \sim 40,000 \]

Which is easy to understand from the uncertainty principle.
Theory for Muon \((g - 2)\)

- First and second order weak:
  - \(+38.9\)
  - \(-19.4\)

- Higher order terms:
  - \(116.584, 795.7(2) \times 10^{-10}\)
  - \(692.4(6.2) \times 10^{10}\)
  - \(-10.1(6) \times 10^{10}\)
  - \(8.5(4.0) \times 10^{-10}\)

- First + Second Order Weak:
  \(-15.1(4) \times 10^{10}\)
Theory for Muon \((g - 2)\), ctd.

\[ a_\mu (\text{SM}) = a_\mu (\text{QED}) + a_\mu (\text{hadronic}) + a_\mu (\text{weak}) \]

\[ a_\mu (\text{New Physics}) = a_\mu (\text{Measured}) - a_\mu (\text{SM}) \]
New Physics Contribution?

- substructure?

\[ \Delta a_\mu = \frac{m_\mu^2}{\Lambda^2} \]

\[ \Lambda \geq 5 \text{ Gev} \]
New Physics Contribution?

- substructure?
  \[
  \Delta a_\mu = \frac{m_\mu^2}{\Lambda^2}
  \]
  \(\Lambda \geq 5\ \text{Gev}\)

- anomalous gauge boson coupling?
  \[g_\text{W} = 2\ ?\]
  W boson substructure?
Supersymmetry

• $\alpha_\mu$ is sensitive to SUSY with large $\tan \beta$
Supersymmetry

• $a_{\mu}$ is sensitive to SUSY with large $\tan \beta$
• toy model with equal $\tilde{m}$ and large $\tan \beta$:

$$a_{\mu}(\text{SUSY}) \sim 150 \times 10^{-11} \tan \beta \left( \frac{100 \text{ GeV}}{\tilde{m}} \right)^2$$

$$\sim 1.31 \text{ ppm} \quad \tan \beta \left( \frac{100 \text{ GeV}}{\tilde{m}} \right)^2$$
The Experimental Technique

Protons from AGS
Target
Pions $\pi^+ \rightarrow \mu^+ \nu_\mu$

$x_c \approx 77 \text{ mm}$
$\beta \approx 10 \text{ mrad}$
$B \cdot dl \approx 0.1 \text{ Tm}$
Beam Requirements at AGS

- For \((g - 2)\): AGS runs at 12th harmonic with 12 proton bunches.
- Fast Extraction: “one bunch at a time”
  Bunch spacing \(\Delta t = 220\) ns.
- Extraction kicker fires at 33 ms intervals.
- AGS Flat-top while 12 bunches are being extracted.

At JHF we would need \(h = 18\) or \(h = 27\) with “one bunch at a time” extraction for \((g - 2)\) or EDM experiments.
Spin and Momentum Precession

\[ \omega_C = \frac{eB}{mc\gamma} \quad \omega_S = \frac{geB}{2mc} + (1 - \gamma) \frac{eB}{\gamma mc} \]

\[ \omega (\vec{S} \text{ relative to } \vec{p}) \quad \omega_a = \omega_S - \omega_C = \left(\frac{g - 2}{2}\right) \frac{eB}{mc} \]

Spin Motion in \( \vec{E} \) and \( \vec{B} \) Fields.

\[ \vec{\omega}_a = \frac{d\Theta_R}{dt} = \frac{e}{mc} \left[ a_{\mu} \vec{B} - \left( a_{\mu} - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right] \]

for \( \gamma = 29.3 \)

\[ \left( a_{\mu} - \frac{1}{\gamma^2 - 1} \right) = 0 \]

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Spin and Momentum Precession

In a uniform $\vec{B}$ field all muons precess at the same rate.

\[ \omega_a = a_\mu \frac{eB}{mc} \]

(exaggerated ~20x)
The Storage Ring Magnet
## Storage Ring Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g-2) Frequency</td>
<td>$f_a \approx 0.23 \times 10^6$ Hz</td>
<td>$\tau_a = 4.37 \mu s$</td>
</tr>
<tr>
<td>Muon kinematics</td>
<td>$p_\mu = 3.094$ GeV/c</td>
<td>$\gamma_\mu = 29.3$</td>
</tr>
<tr>
<td></td>
<td>$\gamma \tau = 64.4$ $\mu s$</td>
<td></td>
</tr>
<tr>
<td>Cyclotron Period</td>
<td>$\tau_{cyc} = 149$ ns</td>
<td></td>
</tr>
<tr>
<td>Central Radius</td>
<td>$\rho = 7112$ mm</td>
<td>$(280''$)</td>
</tr>
<tr>
<td>$B_0 = 1.451$ T</td>
<td>Storage Aperture</td>
<td>9.0 cm circle</td>
</tr>
<tr>
<td>In one lifetime:</td>
<td>432 revolutions around ring</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$14.7$ (g-2) periods</td>
<td></td>
</tr>
</tbody>
</table>
\[< B > = \langle \int M(r, \theta)B(r, \theta)rd\theta \rangle \phi\]

366 fixed NMR probes monitor field stability.
And $< B >_\phi$ is:

In 1999 Quadrupole $\simeq 2.0$ ppm of $B_0$

in 2000 $\simeq 0.2$ ppm of $B_0$. Perfect for NuMass!
The $(g - 2)$ Detector Geometry

- muon momentum
- Sci-Fi Calorimeter module
- Measures Energy and time
- spin forward, more high energy e
- spin backward, less high energy e

400 MHz digitizer
Coherent Betatron Frequency

\[ f_{\text{CBO}} = f_C - f_x = (1 - \sqrt{1 - n}) f_C \]

\( (\lambda_{\text{CBO}} \sim 14 \text{ turns}) \)

\( f_{\text{CBO}} \) amplitude modulates the \( e^\pm \) signal.
Time Spectrum, $E > 2.0 \text{GeV}$

$\sigma_{\text{stat}} \approx 0.7 \text{ ppm}$

$$f(t) = N_0 e^{-\lambda t} [1 + A \cos(\omega_\alpha t + \phi)]$$
\[ \omega_a = \frac{e}{m} a_\mu < B > \]

Remove offsets and divide to determine

\[ R = \frac{\omega_a}{\omega_p} \]

From

\[ a_\mu = \frac{R}{\lambda - R} \quad \lambda = \frac{\mu_\mu}{\mu_p} \]

Add corrections for radial \( \vec{E} \)-field and vertical “pitching motion”. \( (+0.81 \pm 0.08 \text{ ppm}) \)

\[ a_{\mu^+} = 11\ 659\ 204(7)(5) \times 10^{-10} (\pm 0.7 \text{ ppm}) \]
Measurements of $a_\mu$

- BOSTON UNIVERSITY

The diagram illustrates measurements of $a_\mu$ and compares them with theoretical predictions from the BOU experiment.

- CERN experiments (97, 98, 99) provide measurements of $\mu^+$.
- E821 (97, 98, 99) experiments provide measurements of $\mu^+$.
- Theory (DH98) predictions are shown below.

The values are:
- 13 ppm
- 5 ppm
- 1.3 ppm
- 0.7 ppm
- Comparisons with experimental results are indicated.

The diagram visually represents the differences between experimental and theoretical values.

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Measurements of $a_\mu$
Muon \((g-2)\) is sensitive to large mass physics!

- If we take the DEHZ \(e^+e^-\) hadronic contribution, and subtract the electroweak contribution from the theory prediction,

\[
\alpha_\mu(\text{QED + Had}) - \alpha_\mu(\text{Exp}) = (49.7 \pm 10.6) \times 10^{-10} \quad 4.6\sigma
\]

We see the \(W\) and \(Z\) contributions at a high level of confidence.
Muon $(g-2)$ is sensitive to large mass physics!

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We see the $W$ and $Z$ contributions at a high level of confidence.

- If we take the full standard model using DEHZ $e^+e^-$

$$a_\mu(\text{SM(DEHZ)}) - a_\mu(\text{Exp}) = (33.9\pm 10.6) \times 10^{-10} \quad 3.2\sigma$$
What if SUSY were true and we knew the Masses?

- Then the SUSY contribution to $(g - 2)$ would become part of the “new standard model”.
What if SUSY were true and we knew the Masses?

- Then the SUSY contribution to \((g - 2)\) would become part of the “new standard model”.
- The measurement of \((g - 2)\) would provide one of the cleanest measurements of \(\tan \beta\).
If the difference with theory means non-SM physics:

There should be an electric dipole moment produced by this same non-standard model physics. (⇒ a new $CP$-violation)

<table>
<thead>
<tr>
<th>Particle</th>
<th>Present EDM Limit (e-cm)</th>
<th>Standard Model Value (e-cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>$6.3 \times 10^{-26}$</td>
<td>$10^{-31}$</td>
</tr>
<tr>
<td>$e^-$</td>
<td>$\sim 1 \times 10^{-27}$</td>
<td>$10^{-38}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$&lt; 10^{-18}$ (CERN)</td>
<td>$\sim 10^{-35}$ Estimated</td>
</tr>
<tr>
<td></td>
<td>$\sim 10^{-19}$ (E821)</td>
<td>Dedicated Experiment</td>
</tr>
<tr>
<td></td>
<td>$\sim 10^{-24}$ New</td>
<td></td>
</tr>
</tbody>
</table>
Experimental Observation of an EDM

- The spin would rise out of the orbit plane with time.
Experimental Observation of an EDM

- The spin would rise out of the orbit plane with time.
- This can be seen from the spin precession formulae with a magnetic and an electric dipole moment.
Spin Precession with EDM

\[ \ddot{\omega} = -\frac{e}{m} \left[ a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1}) \vec{\beta} \times \vec{E} \right] \]

\[ + \frac{e}{m} \left[ \frac{\eta}{2} \left( \frac{\vec{E}}{c} + \vec{\beta} \times \vec{B} \right) \right] \]

\[ d_\mu = \frac{\eta}{2} \left( \frac{e\hbar}{2mc} \right) \approx \eta \times 4.7 \times 10^{-14} \quad e \text{ cm} \]

and

\[ a_\mu = \left( \frac{g - 2}{2} \right) \]
Vector relationship for $\omega_a, \omega_\eta$

(not to scale)

The EDM causes the spin to precess out of the plane!
If we could turn off $\omega_a$

e.g. with a radial $\vec{E}$ field, the spin would rise monotonically with time.
In SUSY

Feng and Matchev estimate that with a deviation of the order seen in \((g - 2)\), the maximum edm expected would be

\[ d_\mu^{\text{max}} \sim 10^{-20} \text{ e cm} \]

The goal of a dedicated experiment would be \(10^{-24} \text{ e cm}\) which gives a substantial range for discovery!
Conclusions and Outlook, $(g - 2)$

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- The final answer will provide an important constraint on new theories, and a future measurement would improve things further.
- The theory will also improve further.
- Technically one could improve things almost an order of magnitude beyond E821.
Conclusions and Outlook, ctd.

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Come join us to do it!