PY231 FINAL EXAMINATION SOLUTIONS - 12 May 1994

1. Wave phenomena are central to musical acoustics.
   1a). Tell how diffraction is important for the radiation of sound from a woodwind instrument. How is this different from a Brass instrument. Tell why a brass band sounds different outdoors when it is facing you and when it is facing away.

Diffraction is the spreading out of a wave if it goes through an opening which is on the order of the wavelength or less, or if it impenges on an object of dimension on the order of the wavelength. In a woodwind, the vibrating air column terminates at the open tone hole, (or just beyond it) so the sound is radiated from the holes, not from the bell of the instrument. Since the holes are much smaller than all but the very shortest wavelengths, the sound is highly diffracted and thus spreads fairly evenly in all directions throughout the room. On the other hand, brass instruments have only one opening, the bell, so all the sound has to come out there.

In a brass instrument, the bell size is considerably larger than the tone holes on a woodwind, so there is less diffraction, and the higher frequencies are quite directional, while the lower ones are diffracted. Thus in a complex sound from a brass instrument, the fundamental and first few harmonics will be dispersed and the treble part will not be, thus changing the timbre of the sound depending on whether the player faces you or not outdoors. Orchestral brass players use this effect too when they play into the stand and floor, rather than the usual way with the bell up facing the audience.

   1b). Briefly describe the issues of room acoustics, emphasizing the importance of reflections and diffraction in the sound which reaches the listener. The normal modes of the room play an essential rol in the room's response to sound. Be sure and include this fact in your disucssion.

After a sound leaves a source, it is spread through the room by diffraction from objects in the room (including the heads of the audience) and also by reflections from the walls. The energy in the sound waves will serve to excite the modes of the air in the room, so that the reverberant sound will build up to a maximum value. It will then decay after the sound is terminated. This is the same thing which happens with any system which is driven. When the driving force is turned on, it takes some time for the oscillations to build up and for the transients to die out. After the driving force is turned off, it takes some time for the motion to stop, and the rate at which each mode dies out depends on its own particular halving time.

   1c). Reflection is important in the forming of standing waves. How is a standing wave formed? For the following examples, what is the wavelength of the lowest frequency standing wave, in terms of the length of the string or pipe. i.) a string of length fixed at both ends, ii.) a cylindrical pipe open at one end and closed at the other, iii.) a cylindrical pipe open at both ends.

A standing wave is formed by the superposition of two waves of the same frequency and wavelength, travelling in opposite directions. In all the musical systems we studied, a disturbance travels down a medium (string or air column) and is reflected back. If the frequency is such that exactly an even number of half wavelengths (or 1/4, 1/2, 3/4 etc) fit in – (which fraction depends on the boundary conditions at the end) then a standing wave can be set up. i). String fixed at both ends. We have a node at each end so the longest wavelength possible is if $\lambda/2 = L$ and $\lambda = 2L$; ii) Pipe open at one end. We have a pressure node at the open end and a pressure anti-node at the closed end. The longest wavelength possible is when $\lambda/4 = L$ so $\lambda = 4L$ iii)pipe open at both ends has a pressure node at each end, so it looks just like a string fixed at each end and $\lambda/2 = L$

2a). Why is a flute an octave above the clarinet, although their lengths are almost the same?

The flute is essentially a cylindrical tube open at both ends so $f_1 = v/\lambda_1$ with $\lambda_1 = 2L$, the clarinet is essentially a cylindrical tube closed at one end (reed end). Thus $\lambda_1 = 4L$ which means that $f_{1\ clarinet} = 1/2 f_{1\ flute}$ (P8 lower).

2b). Why does the lower register of the clarinet sound so much different from other wind instruments?

A tube closed at one end has mode frequencies which are harmonically related, but only the odd harmonics. Thus the puffs of air can only excite modes which have frequencies f, $3f$, $5f$... and the resulting sound is missing the even harmonics.

2c). Why does a guitar string plucked at its center sound so different?

Only odd numbered modes are excited, since all even numbered modes have a node at the midpoint of the string, so the partials produced by plucking exactly in the middle consist of frequencies which are almost harmonically related, but only the odd harmonics.

2.d) Tell what the acoustic guitar player can do to change the timbre of the sound of the instrument. Tell why each of the techniques or devices you list works. Be sure and explain why vibration of the string perpendicular to the plane of the guitar front plate will decay (sound) for a much shorter time than vibration in the plane parallel to the plate.

Change the plectrum (soft or hard pick, finger, fingernail) change the position closer or further from bridge. The hard plectrum will excite more of the higher modes than a softer one. Plucking closer to the bridge will also excite the higher modes. Since the bridge impedance is less for vibrations where the string is moving perpindicular to the plane of the top plate, than for motion where the string is moving in a plane parallel to the plate, that portion of the motion will die out faster and have a more brilliant percusive quality.

3. Impedance is an important concept in musical acoustics. At some place in an instrument there is an impedance mismatch which causes a large fraction of a wave to be reflected back. In the instruments which produce steady sounds, the result is a standing wave. Discuss the role of impedance in the piano, guitar and the plucked violin. Describe what factors are important in the ringing of the sound after the string is struck or plucked. What would happen to the piano sound if the three strings of the same pitch were exactly in tune? Why is the short decay time of the plucked violin string acceptable musically, whereas it would not be for the guitar, piano or harp, (hint, how is it usually played) and how does this help the dynamic range of the bowed instrument (help the sound to radiate into the air).

In the guitar, piano and violin, the impedance of the bridge is greater than that of the string, so much of the wave in the string will be reflected when it arrives at the bridge. The transmitted wave goes on to drive the soundboard, thus driving the air and permitting us to hear the sound. The decay time of the piano, guitar, harp is determined by how long it takes the energy in the vibrating string to be dissipated. The impedance mismatch determines how much of the wave (and thus energy) is reflected each time as the wave travels up and down the string, and thus how long the sound lasts. If all three stings on a piano are vibrating in step, the effective impedance will be three times higher, and the decay time will be three times faster.

For the violin, which is usually driven by a bow, the impedance mismatch at the bridge is less than for the others, thus more energy goes into the body to drive the air. A plucked string on a violin decays quite fast. The result is that a much larger dynamic range is possible for a violin than for the guitar. The piano has such a large dynamic range too, but this is because you can strike the strings with the hammers much harder than you can pluck a guitar or harpsicord string.

4. Standing waves are essential to musical instruments' performance. Using either the violin, flute, clarinet or trumpet as an example, sketch the first three modes of the string or air column. (Caution, the trumpet is not a simple cylindrical tube). For the air column case, please use the pressure and not displacement. Tell why it is important that the modes of the instrument have harmonically related frequencies. Identify the nonlinearity which generates the harmonic spectrum in the driving force for the clarinet, and violin. On your sketch, be sure and indicate the relationship between the frequencies of the higher modes and the lowest one, i.e. assume that the lowest has frequency f, give the higher mode frequency in terms of f.

Harmonically related mode frequencies: The player must excite higher modes of the instrument as well as the fundamental to produce a musically acceptable sound. The driving force always has harmonics of the driving (fundamental) frequency because of some characteristic non- linearity. If these higher frequencies present in the driving force are to excite modes, these modes must have frequencies close to harmonics of the fundamental, or they will not be excited. The nonlinearity in the violin is the nonlinear force between the string and the bow as a function of the relative velocity between the string and the bow. In the clarinet it is the fact that the air which flows through an opening is not proportional to the opening size, but depends on the opening size in a non-linear way.

5. The harmonic series lies at the center of many of the topics we have discussed this semester, including the tuning of intervals and the building of musical scales.

5a). Please list the simplest sum and difference notes generated by a P5 ($C_4 \rightarrow G_4$) P4 ($G_4 \rightarrow C_5$) and a M3 ($C_5 \rightarrow E_5$). Hint, remember how to use the harmonic series to answer this question.

The harmonic series based on C_2 *is*
C_3 C_4 G_4 C_5 E_5 G_5 $B\flat_5$ C_6 D_6 E_6...
where I have listed the first 10 harmonics. (the 7th is flat). Now for sum and difference tones, we have the sum tone $f_1 + f_2$*, and the difference tone* $f_1 - f_2$ *(where we ignore any negative signs we get with the difference). Since harmonics have frequencies related by integers, we can just use the harmonic numbers to get the note which corresponds to a sum or difference. The* $P5$ *is the interval between the second and third harmonics, so* $3+2=5$*, i.e.* C_4, G_4 *so* E_5 *is sum and* C_3 *is the difference. Note also that for* $P4$ *and* $M3$ *we get the first harmonic* C_3 *for the difference tone in each case. The sum tones are* $4+3=7$ $B\flat_5$ *and* $4+5=9$ D_6 *respectively.*

5b). Beat free intervals are those formed by two musical sounds which have simple integer ratios between the fundamental frequencies of the harmonic series of the sounds. We have discovered that it is impossible to construct a scale of beat free notes. Give the advantages of equal temperament, as well as its disadvantages. What are the limitations of the just scale, which permits one to have the three important triads, tonic, dominant and sub-dominant beat free. (You should tell why a contemporary (non rock and roll) composer would find this very constraining, and why, even a rock and roll composer might find it limiting.

Equal temperament makes all intervals the same number of cents, so tuning errors are distribuited throughout the chromatic scale. Only P8 are in tune. All keys are in principle equivalent, and P5, P4 M3 etc. are all not beat free. This permits the composer to use a wide range of notes, keys, to modulate from one to the other, all without having to re-tune the instruments. The just scale permits one to play beat free intervals in a home key, at least for the important triads mentioned above, however these are not the only possible chords, even in a given key, and the ability to modulate to other keys is severly limited without *having to retune. Primitive rock and roll (or the Blues) was largely restricted to 1/4/5 chord changes, however, even that music occasionally changed keys. Certainly from one song to the next, it was desirable to change keys, so the just scale would not be so useful there either. Only in the early music, really before Bach, was the just scale useful. Once Bach showed how to hide the tuning errors, especially in the M3, musical composition left this simpler world behind and has not looked back.*