Hadronic Vacuum Polarization

Confronting Spectral Functions from $e^+e^-$ Annihilation and $\tau$ Decays: Consequences for the Muon Magnetic Moment

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LAL - Orsay

Lepton Moments
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Outline

The Muon Magnetic Anomaly
... and Hadronic Vacuum Polarization

Isospin Breaking
and Radiative Corrections

$e^+e^-$ versus $\tau$ Data

Standard Model + Data Predictions for $(g-2)_\mu$

$\alpha_\mu[\text{had}]$

$\Delta \alpha_{\text{QED}}[\text{had}]$
Magnetic Anomaly

QED Prediction:
Computation up to 4th order
[Kinoshita et al.]
(5th order estimated
[Mohr, Taylor])

\[ \Gamma_\mu = e \gamma_\mu + a_\mu \frac{ie}{2m} \sigma_{\mu\nu} q_\nu \]

\[ a_\mu = \frac{\alpha}{2\pi} = 0.001161\ldots \]

\[ a_\mu^{QED} = \sum_{n=1}^{\infty} \left( \frac{\alpha}{\pi} \right)^n \approx \left( 11614098.1 + 41321.8 \right) + \left( 3014.2 + 36.7 + 0.6 \right) \times 10^{-10} \]

QED: Hadronic: Weak: SUSY...

... or other new physics?
Why Do We Need to Know it so Precisely?

The fine structure constant at $M_Z$ is an important ingredient to EW precision fits.

Experimental Progress on Precision of $(g-2)_\mu$

Aiming at:

$$\sigma_{\exp}(a_\mu) \approx 4 \times 10^{-10}$$

$$\approx \sigma(a_\mu^{\text{had}})$$

$$\ll a_\mu^{\text{weak}} \text{ [SM]}$$
The Muonic \((g-2)\)\(\mu\)

Contributions to the Standard Model (SM) Prediction:

\[
a_\mu \equiv \left( \frac{g-2}{2} \right) = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{weak}}
\]

<table>
<thead>
<tr>
<th>Source</th>
<th>(\alpha(\mu))</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
<td>(\sim 0.3 \times 10^{-10})</td>
<td>[Schwinger ’48 &amp; others]</td>
</tr>
<tr>
<td>Quarks and Hadrons</td>
<td>(\sim (15 \oplus 4) \times 10^{-10})</td>
<td>[Eidelman-Jegerlehner ’95 &amp; others]</td>
</tr>
<tr>
<td>(Z, W) exchange</td>
<td>(\sim 0.4 \times 10^{-10})</td>
<td>[Czarnecki et al. ‘95 &amp; others]</td>
</tr>
</tbody>
</table>

Dominant uncertainty from lowest order hadronic piece. Cannot be calculated from QCD (“first principles”) – but: we can use experiment (!)

\[
a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2} \int ds \frac{K(s)}{s} R(s)
\]

"Dispersion relation"
Data density and quality unsatisfactory in some crucial energy regions.
Improved Determination of the Hadronic Contribution to $(g-2)_\mu$ and $\alpha(M_Z^2)$

Situation 1995 [Eidelman-Jegerlehner] and since ...

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$2m_\pi - 1.8$</td>
<td>Data</td>
<td>Data (e$^+e^-$ &amp; $\tau$) + QCD</td>
</tr>
<tr>
<td>$1.8 - J/\psi$</td>
<td>Data</td>
<td>QCD</td>
</tr>
<tr>
<td>$J/\psi - \gamma$</td>
<td>Data</td>
<td>Data + QCD</td>
</tr>
<tr>
<td>$\gamma - 40$</td>
<td>Data</td>
<td>QCD</td>
</tr>
<tr>
<td>$40 - \infty$</td>
<td>QCD</td>
<td>QCD</td>
</tr>
</tbody>
</table>

Improvement in 3 Steps:

- **Inclusion of precise $\tau$ data using SU(2) (CVC)**
  - Alemany-Davier-Hocker’97, Narison’01, Trocóniz-Ynduráin’01

- **Extended use of (dominantly) perturbative QCD**
  - Martin-Zeppenfeld’95, Davier-Hocker’97, Kühn-Steinhauser’98, Erler’98, + others

- **More theoretical constraints from QCD sum rules**
  - Groote, Körner, Schilcher, Nasrallah’98, Davier-Hocker’98, Martin-Outhwaite-Ryskin’00, Cvetič-Lee-Schmidt’01, + others
A New Analysis of $a_{\mu}^{\text{had}}$

Motivation for new work:
- **New high precision $e^+e^-$ results (0.6% sys. error)** around $\rho$ from CMD-2 (Novosibirsk)
- **New $\tau$ results** from ALEPH using full LEP1 statistics
- **New $R$ results** from BES between 2 and 5 GeV
- **New theoretical analysis of SU(2) breaking**

Outline of the new analysis:
- Include all new Novosibirsk (CMD-2, SND) and ALEPH data
- Apply (revisited) SU(2)-breaking corrections to $\tau$ data
- Identify application/non-application of radiative corrections
- Recompute all exclusive, inclusive and QCD contributions to dispersion integral; revisit threshold contribution and resonances
- Results, comparisons, discussions...

References:
- ALEPH CONF 2002-19
- BES PRL 84 594 (2000); PRL 88, 101802 (2002)
- Cirigliano-Ecker-Neufeld hep-ph/0207310
The Conserved Vector Current – SU(2)

Hadronic physics factorizes in **Spectral Functions**:

Isospin symmetry (CVC) connects $I=1$ $e^+e^-$ cross section to vector $\tau$ spectral functions:

$$\sigma^{(I=1)}[e^+e^- \rightarrow \pi^+\pi^-] = \frac{4\pi\alpha^2}{s} \nu[\tau^- \rightarrow \pi^-\pi^0\nu_\tau]$$

$$\nu[\tau^- \rightarrow \pi^-\pi^0\nu_\tau] \propto \frac{\text{BR}[\tau^- \rightarrow \pi^-\pi^0\nu_\tau]}{\text{BR}[\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau]} \frac{1}{N_{\pi\pi^0}} \frac{dN_{\pi\pi^0}}{ds} \frac{m^2_{\tau}}{(1-s/m^2_{\tau})^2(1+s/m^2_{\tau})}$$

branching Fractions  mass spectrum  kinematic factor (PS)
Spectral Functions

Hadronic $\tau$ decays are a clean probe of hadron dynamics – experimentally in many ways complementary to $e^+e^- \rightarrow$ hadrons:

- Excellent normalization (branching fractions) due to high statistics, large acceptance, small non-$\tau$ background
- Shape subject to bin-to-bin resolution corrections (unfolding)
- Good relative cross sections (correlated systematics, but large rad. corr.)
- Overall normalization subject to radiative corrections, systematic uncertainties from acceptance and luminosity

![Graphical representation of $\tau \rightarrow (V^-, I=1) \nu_\tau$ (ALEPH)](ALEPH ZP C76, 15 (1997))

![Graphical representation of $\tau \rightarrow (A^-, I=1) \nu_\tau$ (ALEPH)](ALEPH EPJ C4, 409 (1998))
QCD Results from $\tau$ Decays

Evolution of $\alpha_s(m_\tau)$, measured using $\tau$ decays, to $M_Z$ using RGE (4-loop QCD $\beta$-function & 3-loop quark flavor matching) shows the excellent compatibility of $\tau$ result with EW fit:

$\alpha_s(M_Z) = 0.1202 \pm 0.0027$ (ALEPH'98, theory dominated)
$\alpha_s(M_Z) = 0.1183 \pm 0.0027$ (LEP’00, statistics dominated)

The $\tau$ spectral function allows to directly measure the running of $\alpha_s(s_0)$ within $\sqrt{s_0} \in [-1...1.8 \text{ GeV}]$
\[ \tau \rightarrow \pi^- \pi^0 \nu_\tau \] Comparing ALEPH, CLEO, OPAL

Good agreement observed between ALEPH and CLEO
- ALEPH more precise at low \( s \)
- CLEO better at high \( s \)

Shape comparison only. Both normalized to WA branching fraction (dominated by ALEPH).
SU(2)Breaking

Electromagnetism does not respect isospin and hence we have to consider isospin breaking when dealing with an experimental precision of 0.5%

Corrections for SU(2) breaking applied to $\tau$ data for dominant $\pi^-\pi^+$ contrib.: 

- Electroweak radiative corrections:
  - dominant contribution from short distance correction $S_{EW}$ to effective 4-fermion coupling $\propto (1 + 3\alpha(m_\tau)/4\pi)(1+2\langle Q\rangle)\log(M_Z/m_\tau)$
  - subleading corrections calculated and small
  - long distance radiative correction $G_{EM}(s)$ calculated
    [ add FSR to the bare cross section in order to obtain $\pi^-\pi^+(\gamma)$ ]

- Charged/neutral mass splitting:
  - $m_{\pi^-} \neq m_{\pi^0}$ leads to phase space (cross sec.) and width (FF) corrections
  - $m_{\rho^-} \neq m_{\rho^0}$ and $\rho - \omega$ mixing (EM $\omega \to \pi^-\pi^+$ decay) corrected using FF model

- Electromagnetic decays, like: $\rho \to \pi\pi\gamma$, $\rho \to \pi\gamma$, $\rho \to \eta\gamma$, $\rho \to ll\gamma$

- Quark mass difference $m_u \neq m_d$ generating “second class currents” (negligible)
Mass Dependence of SU(2) Breaking

Multiplicative SU(2) corrections applied to $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ spectral function:

Only $\beta^3$ and EW short-distance corrections applied to $4\pi$ spectral functions
### SU(2) Breaking

#### Corrections for isospin violation applied to $\tau$ data

<table>
<thead>
<tr>
<th>Source</th>
<th>$\pi^-\pi^+$ (simple)</th>
<th>$\pi^-\pi^+$ (improved)</th>
<th>$\pi^-\pi^+ \ 2\pi^0$</th>
<th>$2\pi^-2\pi^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short distance radiative corrections to $\tau$ decays (for $S_{EW} = 1.0233 \pm 0.0006$) [Marciano-Sirlin'88, Braaten-Li'90, new evaluation DEHZ'02]</td>
<td>$-10.3 \pm 2.1$</td>
<td>$-12.1 \pm 0.3$</td>
<td>$-0.36 \pm 0.07$</td>
<td>$-0.18 \pm 0.04$</td>
</tr>
<tr>
<td>Long distance corrections</td>
<td>-</td>
<td>$-1.0$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$m_{\pi^-} \neq m_{\pi^0}$ ($\beta$ in cross section)</td>
<td>$-7.0$</td>
<td>$-7.0$</td>
<td>$+0.6 \pm 0.6$</td>
<td>$-0.4 \pm 0.4$</td>
</tr>
<tr>
<td>$m_{\pi^-} \neq m_{\pi^0}$ ($\beta$ in $\rho$ width)</td>
<td>$+4.2$</td>
<td>$+4.2$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$m_{\rho^-} \neq m_{\rho^0}$ ($\pm \approx 1$ MeV/c²)</td>
<td>$+0 \pm 0.2$</td>
<td>$+0 \pm 2.0$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho^-\omega$ mixing (exp. uncertainty)</td>
<td>$+3.5 \pm 0.6$</td>
<td>$+3.5 \pm 0.6$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>EM decay modes</td>
<td>$-1.4 \pm 1.2$</td>
<td>$-1.4 \pm 1.2$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Total correction</td>
<td>$-11.0 \pm 2.5$</td>
<td>$-13.8 \pm 2.4$</td>
<td>$+0.2 \pm 0.6$</td>
<td>$-0.6 \pm 0.4$</td>
</tr>
</tbody>
</table>
**e^+e^- Radiative Corrections**

Multiple radiative corrections are applied on measured e^+e^- cross sections.

**Situation often unclear: whether or not - and if - which corrections were applied**

- **Vacuum polarization (VP) in the photon propagator:**
  - leptonic VP in general corrected for
  - hadronic VP correction not applied, but for CMD-2 (in principle: iterative proc.)

- **Initial state radiation (ISR)**
  - corrected by experiments

- **Final state radiation (FSR) [we need e^+e^- → hadrons (γ) in dispersion integral]**
  - mostly, experiments obtain bare cross section so that FSR has to be added “by hand”; done for CMD-2, (supposedly) not done for others
Comparing $e^+e^- \rightarrow \pi^+\pi^-$ and $\tau \rightarrow \pi^-\pi^0\nu_\tau$

Remarkable agreement

But: is it good enough?

Correct $\tau$ data for missing $\rho$-$\omega$ mixing (taken from BW fit) and all other SU(2)-breaking sources
Comparing $e^+e^- \rightarrow \pi^+\pi^-$ and $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$

Relative difference between $\tau$ and $e^+e^-$ data

Zoom
Comparing the $4\pi$ Spectral Functions

Large discrepancies between experiments

CVC relations (from isospin rotation):

$$\sigma^{(l=1)}_{2\pi^+2\pi^-} = 2 \cdot \frac{4\pi\alpha^2}{s} \nu_{\pi^-3\pi^0\nu_\tau}$$

$$\sigma^{(l=1)}_{\pi^+\pi^-2\pi^0} = \frac{4\pi\alpha^2}{s} \left( \nu_{2\pi^-\pi^+\pi^0\nu_\tau} - \nu_{\pi^-3\pi^0\nu_\tau} \right)$$
Testing CVC

Infer $\tau$ branching fractions from $e^+e^-$ data:

$$\text{BR}_{\text{CVC}}(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) = \frac{6\pi |V_{ud}|^2 S_{EW}}{m_\tau^2} \int_0^{m_\tau^2} ds \ \text{kin}(s) \cdot \nu^{\text{SU(2)-corrected}}(s)$$

Difference: $\Delta(\tau - e^+e^-)_{\text{CVC}}$

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\Delta(\tau - e^+e^-)$</th>
<th>$\text{&quot;Sigma&quot;}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$</td>
<td>$+1.48 \pm 0.32$</td>
<td>4.6</td>
</tr>
<tr>
<td>$\tau^- \rightarrow \pi^- 3\pi^0 \nu_\tau$</td>
<td>$-0.08 \pm 0.11$</td>
<td>0.7</td>
</tr>
<tr>
<td>$\tau^- \rightarrow 2\pi^- \pi^+ \pi^0 \nu_\tau$</td>
<td>$+0.91 \pm 0.25$</td>
<td>3.6</td>
</tr>
</tbody>
</table>
Specific Contributions: Low s Threshold

Use Taylor expansion above $\pi^+\pi^-$ threshold:

$$\sigma_{\pi\pi} = \frac{\pi\alpha^2 \beta^3}{3s} |F_{\pi}|^2$$

and:

$$F_{\pi} = 1 + \frac{1}{6} \left< r^2 \right>_{\pi} s + c_1 s^2 + c_2 s^3 + O(s^4)$$

- exploiting precise space-like data, $\left< r^2 \right>_{\pi} = (0.439 \pm 0.008)$ fm$^2$, and fitting $c_1$ and $c_2$

Fit range:
- up to 0.35 GeV$^2$

Integration range:
- up to 0.25 GeV$^2$

- Excellent $\chi^2$ for both $\tau$ and $e^+e^-$ data
- strong anti-correlation between $c_1$ and $c_2$
Specific Contributions: the Charm Region

New precise BES data improve $c\bar{c}$ resonance region:

Agreement among experiments
Specific Contributions: Narrow Resonances

Use direct data integration for $\omega (782)$ and $\phi (1020)$ to account for non-resonant contributions. However, careful integration necessary:

- trapezoidal rule creates systematics for functions with strong curvature
- use phenomenological fit

---

Trapezoidal rule creates bias

SND, PRD, 072002 (2001)
Results: the Data & the Theory

- Agreement between Data (BES) and pQCD
- Better agreement between exclusive and inclusive ($\gamma\gamma$2) data than in previous analysis
## Results: the Compilation

**Contributions to \( a_{\mu}^{\text{had}} \) from the different energy domains:**

<table>
<thead>
<tr>
<th>Modes</th>
<th>Energy range [GeV]</th>
<th>( e^+e^- )</th>
<th>( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Low s expansion</strong></td>
<td>( 2m_\pi - 0.5 )</td>
<td>( 58.0 \pm 1.7 \pm 1.1_{\text{rad}} )</td>
<td>( 56.0 \pm 1.6 \pm 0.3_{\text{SU(2)}} )</td>
</tr>
<tr>
<td>( \pi^+\pi^- )</td>
<td>( 2m_\pi - 1.8 )</td>
<td>( 440.8 \pm 4.7 \pm 1.5_{\text{rad}} )</td>
<td>( 464.0 \pm 3.2 \pm 2.3_{\text{SU(2)}} )</td>
</tr>
<tr>
<td>( \pi^+\pi^-2\pi^0 )</td>
<td>( 2m_\pi - 1.8 )</td>
<td>( 16.7 \pm 1.3 \pm 0.2_{\text{rad}} )</td>
<td>( 21.4 \pm 1.4 \pm 0.6_{\text{SU(2)}} )</td>
</tr>
<tr>
<td>( 2\pi^+2\pi^- )</td>
<td>( 2m_\pi - 1.8 )</td>
<td>( 14.0 \pm 0.9 \pm 0.2_{\text{rad}} )</td>
<td>( 12.3 \pm 1.0 \pm 0.4_{\text{SU(2)}} )</td>
</tr>
<tr>
<td>( \omega (782) )</td>
<td>( 0.3 - 0.81 )</td>
<td>( 36.9 \pm 0.8 \pm 0.8_{\text{rad}} )</td>
<td>-</td>
</tr>
<tr>
<td>( \phi (1020) )</td>
<td>( 1.0 - 1.055 )</td>
<td>( 34.8 \pm 0.9 \pm 0.6_{\text{rad}} )</td>
<td>-</td>
</tr>
<tr>
<td>Other exclusive</td>
<td>( 2m_\pi - 2.0 )</td>
<td>( 32.2 \pm 1.6 \pm 0.3_{\text{rad}} )</td>
<td>-</td>
</tr>
<tr>
<td>( J/\psi, \psi(2S) )</td>
<td>( 3.08 - 3.11 )</td>
<td>( 7.4 \pm 0.4 \pm 0_{\text{rad}} )</td>
<td>-</td>
</tr>
<tr>
<td>( R [\text{data}] )</td>
<td>( 2.0 - 5.0 )</td>
<td>( 33.9 \pm 1.7_{\text{exp}} \pm 0_{\text{rad}} )</td>
<td>-</td>
</tr>
<tr>
<td>( R [\text{QCD}] )</td>
<td>( 5.0 - \infty )</td>
<td>( 9.9 \pm 0.2_{\text{theo}} )</td>
<td>-</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>( 2m_\pi - \infty )</td>
<td>( 684.7 \pm 6.0 \pm 3.6_{\text{rad}} )</td>
<td>( 709.0 \pm 5.1 \pm 1.2_{\text{rad}} \pm 2.8_{\text{SU(2)}} )</td>
</tr>
</tbody>
</table>
Discussion

The problem of the $\pi^+\pi^-$ contribution [shifts given in units of $10^{-10}$]:

- **Experimental conspiracy:**
  - new CMD-2 data produce downward shift [-1.9], with much better precision
  - new ALEPH BRs produce upward shift [+3.5]
  - CLEO & OPAL spectral functions produce upward shift [+5.9]
  - previous difference was: $\Delta[\tau - e^+e^-] = (11 \pm 15) \times 10^{-10}$ → we could use average

- **Who is wrong?**
  - $e^+e^-$ is consistent among experiments, but error dominated by CMD-2; large radiative corrections applied
  - $\tau$ is consistent among experiments, but error dominated by ALEPH
  - SU(2) corrections: basic contributions identified and stable since long; overall correction applied to $\tau$ is $(-2.2 \pm 0.5)\%$, dominated by uncontroversial short distance piece; additional long-distance corrections found to be small

At present, we believe that it is inappropriate to combine $\tau$ and $e^+e^-$:

$$\Delta[\tau - e^+e^-] = (-24.3 \pm 6.9\,\text{exp} \pm 2.7\,\text{rad} \pm 2.8\,\text{SU(2)}) \times 10^{-10}$$
A few checks detailed

\( \pi^+\pi^- \) contribution [in units of 10\(^{-10}\)]:

- **e\(^+\)e\(^-\) consistency (contribution 610 - 820 MeV)**
  - CMD-2: 313.5 ± 3.1
  - CMD: 320.8 ± 12.6
  - OLYA: 321.8 ± 13.9 (rad. corr. and SU(2) breaking not included in errors)

- **\(\tau\) consistency (contribution > 500 MeV)**
  - ALEPH: 460.1 ± 4.4
  - CLEO: 464.7 ± 9.3
  - OPAL: 464.2 ± 8.1

- **Test of ALEPH \(\tau \rightarrow \pi \pi^0 \nu_\tau\) branching fraction**

  global analysis of \(\tau\) decays: \(\pi \leftrightarrow \pi \pi^0 \leftrightarrow \pi \pi^0 \pi^0\)

  - Predicted by \(\tau\)-\(\mu\) universality
  - From \(\pi \pi \pi\) by isospin

  \[ B_{\pi} - B_{\pi}^{\text{uni}} = (-0.08 \pm 0.11_{\text{exp}} \pm 0.04_{\text{th}}) \% \]

  \[ B_{\pi2\pi^0} - B_{\pi2\pi^0}^{3\pi, \text{iso}} = (+0.06 \pm 0.17_{\text{exp}} \pm 0.07_{\text{th}}) \% \]

  excludes a 1.1 % bias in the \(\pi \pi^0\) fraction
Final Results

\[ a^{\text{had}}_\mu [ee] = (684.7 \pm 7.0) \times 10^{-10} \]  
\[ a^{\text{had}}_\mu [\tau] = (709.0 \pm 5.9) \times 10^{-10} \]

\[ \Delta[\tau - e^+e^-] = (3.5 \pm 1.1)\% \]

\[ (11659169.3 \pm 7.0_{\text{had}} \pm 3.5_{\text{LBL}} \pm 0.4_{\text{QED+EW}}) \times 10^{-10} \]

\[ a^{\mu}_{ee} = (709.0 \pm 5.9) \times 10^{-10} \]

\[ a^{\mu}_{\tau} = (11659193.6 \pm 5.9_{\text{had}} \pm 3.5_{\text{LBL}} \pm 0.4_{\text{QED+EW}}) \times 10^{-10} \]

\[ 692.4 \pm 6.2 \]

Hadronic contribution from higher order:
\[ a^{\text{had}}_\mu [(\alpha_s/\pi)^3] = - (10.0 \pm 0.6) \times 10^{-10} \]

Hadronic contribution from LBL scattering:
\[ a^{\text{had}}_\mu [\text{LBL}] = + (8.6 \pm 3.5) \times 10^{-10} \]

Observed Discrepancy:
\[ a^{\mu}_{\text{[exp]}} - a^{\mu}_{\text{[SM]}} = \begin{cases} 34 \pm 11 \quad [e^+e^-] \\ 9 \pm 11 \quad [\tau] \end{cases} \]

Effect on \( \Delta \alpha_{\text{had}}(M_Z^2) \):
\[ \Delta[\tau - e^+e^-] = (2.8 \pm 0.8) \times 10^{-4} \]
\[ \Rightarrow \Delta M_{\text{Higgs}} \approx 16 \text{ GeV/c}^2 \text{ for } \tau \]
Conclusions/Perspectives

- Hadronic vacuum polarization creates dominant systematics for SM predictions of many precision measurements, such as the muon $g-2$
- New analysis of leading hadronic contribution motivated by new, precise $e^+e^-$ (0.6% systematic error for $e^+e^-$) and $\tau$ (0.5% error on normalization) data
- New theoretical analysis confirmed the rules to correct for SU(2) breaking
- Radiative (VP and FSR) corrections in $e^+e^-$ are major source of systematics
- We have re-evaluated all exclusive and inclusive as well as resonance contributions
- We conclude with two incompatible numbers from $e^+e^-$ and (mainly) $\tau$, leading to SM predictions that differ by $3.0 \sigma [e^+e^-]$ and $0.9 \sigma [\tau]$ from the experiment

- The key problem is the quality of the experimental data...
- Future experimental projects are:
  - CLEO & BES as $\tau$/charm factories
  - B factories: will improve the line shape from $\tau$, but not the normalization
  - ISR production $e^+e^- \rightarrow \text{hadrons} + \gamma$ @ KLOE, BABAR (systematics?)
Preliminary Look into BaBar Analysis
ISR Method at BaBar: $e^+e^- \rightarrow \gamma f$

\[
\frac{d\sigma(s, x)}{dx} = W(s, x)\sigma_f [s(1-x)]; \quad x = \frac{2E_\gamma^*}{\sqrt{s}}
\]

\[
W(s, x) = \beta \cdot \left[ (1+\delta)x^{(\beta-1)} - 1 + \frac{x}{2} \right] \quad (\delta\approx0.067 \text{ at } \Upsilon(4S))
\]

\[
\beta = \frac{2\alpha}{\pi} (2\ln \frac{s}{m_e} - 1) \quad (\approx0.088 \text{ at } \Upsilon(4S))
\]

Cross Section for final state $f$ (normalized to radiative dimuons)

\[
\sigma_f(s') = \frac{dN_{f\gamma}}{dN_{\mu\mu}} \cdot \varepsilon_{\mu\mu} \cdot (1 + \delta_{\mu\mu}^{\text{FSR}}) \cdot \sigma_{e^+e^- \rightarrow \mu^+\mu^- (s')} \cdot dL(s')
\]

Detection efficiencies

Corrections for final state radiation

"effective c.m. energy" = $s(1-x)$

ISR luminosity

$\gamma$ detected at large angle in BaBar
BaBar ISR : $e^+e^- \rightarrow \gamma \mu^+\mu^-$

normalization and important test sample

- measurement of $\mu$ ID efficiency
- kinematic fit
- muon sample purity > 99.9 %
- kin. fit improves resolution: $16 \rightarrow 8$ MeV at J/$\psi$
- muon detection weak point of BaBar
\( dL(M_{\text{inv}}^{\mu\mu}) = \frac{dN_{\mu\gamma}(M_{\text{inv}}^{\mu\mu})}{\varepsilon_{\mu\mu} \cdot (1 + \delta_{\text{FSR}}^{\mu\mu}) \cdot \sigma^{e^+e^-\rightarrow\mu^+\mu^-}(M_{\text{inv}}^{\mu\mu})} \)

**Equivalent e^+e^- luminosity:** BaBar’s 90 fb\(^{-1}\) corresponds to an e^+e^- scan with \(\sim 700\) nb\(^{-1}\)/0.1 GeV at 1 GeV and \(\sim 4\) pb\(^{-1}\)/0.1 GeV at 4 GeV.

\( \rightarrow \text{Statistically very competitive continuum sample} \)
BaBar ISR: $e^+e^- \rightarrow \gamma \mu^+\mu^-$ at $J/\psi$ (1)

$$\sigma_{J/\psi}(s) = \frac{12\pi^2 \Gamma_{ee} B_{\mu\mu}}{m \cdot s} \cdot W(s, x_0), \quad x_0 = (1 - \frac{m^2}{s})$$
BaBar ISR: $e^+e^- \rightarrow \gamma \mu^+\mu^-$ at $J/\psi$ (2)

$$\Rightarrow \Gamma_{ee} \cdot B_{\mu\mu} = 0.3301 \pm 0.0077 \pm 0.0073 \text{ keV}$$

$$B_{\mu\mu} = (5.88 \pm 0.10)\% \quad \Rightarrow \quad B_{ee} = (5.93 \pm 0.10)\%$$

$$\Gamma_{ee} = 5.61 \pm 0.20 \text{ keV}$$

$$\Gamma = 94.7 \pm 4.4 \text{ keV}$$

5.26 ± 0.37 keV

PDG2002:

87 ± 5 keV

Hsueh 1992
E760 1993
BES 1995
PDG 2002
This work
BaBar ISR: $e^+e^- \rightarrow \gamma \pi^+ \pi^-$

$$\frac{N_{\pi\pi}}{N_{\mu\mu}} \propto \left| F_\pi(s) \right|^2$$

Boost:
- acceptance down to threshold
- easier particle ID

Ratio cancels:
- luminosity
- ISR and VP radiative corrections
- many efficiencies (photon, tracking)

Small corrections:
- trigger efficiency (track and EMC triggers)
- FSR corrections, can be studied exp.

Major work: particle ID efficiency matrix ($P, \theta, \phi$)
BaBar ISR: $e^+e^- \rightarrow \gamma \pi^+\pi^-$

- The data correspond to an integrated luminosity of 88 fb$^{-1}$.
- Background from $e^+e^- \rightarrow \gamma \mu^+\mu^-$ events is at the level of 1%.
- The present statistics competes well with the latest results from CMD-2.
- Large mass range coverage
- Work in progress on the control of the systematics, in particular for particle identification.
• higher masses require the study of many channels

• **particle ID essential** (DIRC) for correct mass determination

• work in progress
  \[ K^+K^- , \ p\bar{p} , \ \pi^+\pi^-\pi^0 , \ 4\pi , \ 5\pi , \ 6\pi , \ \pi\pi\eta , \ K\bar{K}\pi , \ K\bar{K}\pi\pi , \ 2K2\bar{K} , \ K\bar{K}\eta \]

• R will be reconstructed from the sum of exclusive channels up to \( \approx 2.5 \text{ GeV} \)

• **inclusive approach** also tried, but limited by photon energy resolution at lower recoil masses
**BaBar ISR**: $e^+e^- \rightarrow \gamma 2\pi^+ 2\pi^-$

- Very clean sample (background~2%)
- Whole mass range is covered
- Large statistics (~75k events)

* Very preliminary: Not finally normalized.
BaBar ISR: $e^+e^- \rightarrow \gamma f$ examples

$K^+K^-$

$K_S K \pi$

$\pi^+ \pi^- \pi^0$

$\omega$

$\phi$

$J/\psi$

$\phi\eta$

$M_{\gamma\gamma}$ (GeV)

$M_{3\pi}$ (GeV)

$M_{\gamma\gamma}$ (GeV)

$M_{3\pi}$ (GeV)

$M_{3\pi}$ (GeV)

$M_{3\pi}$ (GeV)

$M_{3\pi}$ (GeV)

$M_{3\pi}$ (GeV)