Ultra High Precision
with a Muon Storage Ring

The Muon \((g - 2)\) Experiment
at Brookhaven National Laboratory

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BNL AGS E821:

A New Precision Measurement of the Muon $(g - 2)$ Value at the level of 0.35 ppm

Boston University, Brookhaven National Laboratory, Budker Institute of Nuclear Physics - Novosibirsk, Cornell University, Fairfield University, KEK, KVI and Rijksuniversiteit - Groningen, University of Heidelberg, University of Illinois, University of Minnesota, Tokyo Institute of Technology, Yale University
E821 Collaboration (4/02)


B. Lee Roberts, EPAC02, 4 June 2002 – p.3/4
Outline of the Talk

- Brief Introduction to $(g - 2)$, the Motivation and Theory
- Overview of the Experimental Technique
- The Precision Storage Ring Magnet
- Beam Dynamics in the $(g - 2)$ Storage Ring
- Summary and Conclusions
Muon: \((2^{nd} \text{ generation lepton})\)

\[
m_{\mu}c^2 = 105.658\,389(34) \text{ MeV} \\
\tau_\mu = 2.197\,03(4) \mu s
\]

Source: \(\pi^- \rightarrow \mu^- \bar{\nu}_\mu\) \(\Rightarrow\) Weak Decay

Parity Violating Decay \(\Rightarrow\) Polarized Muons

Weak Decay: \(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu\)
Magnetic Moments, \( g \)-Factors, etc.

\[ \vec{\mu}_s = g_s \left( \frac{e}{2m} \right) \vec{s} \]

\( \vec{\mu} \) - magnetic moment; \( g \) - gyromagnetic ratio
\( \vec{s} \) is the spin.

- Dirac Equation Predicts \( g \equiv 2 \)
- In nature radiative corrections make \( g \neq 2 \).

Dirac Equation Predicts \( g \equiv 2 \)

\[ g = 2 + \frac{\alpha}{\pi} + \cdots \]

Kusch and Foley, Schwinger, 1947
Magnetic Moments: Definitions and Values

\[ \mu = (1 + a) \frac{e\hbar}{2m} \]

where \[ a = \left( \frac{g - 2}{2} \right) \]

\[ \mu_e = 1.001\ 159\ 652\ 193 \frac{e\hbar}{2m_e}; \]

For comparison:

\[ \mu_\mu = 1.001\ 165\ 923 \frac{e\hbar}{2m_\mu}; \]

\[ \mu_p = 2.792\ 847\ 39 \frac{e\hbar}{2m_p}; \]

\[ g_p = 5.5857 \cdots \neq 2 \]
Theoretical Value for \((g - 2)\)

- **Electron**: To the level of the experimental error, \(\pm 4\) ppb

\[ a_e(\text{Standard Model}) = a_e(\text{QED with } \gamma, \ e) \]

Contribution of virtual \(\mu, \tau, \text{ etc.}\) is \(\leq 4\) ppb.

- **Muon**: The Relative Contribution of heavier things:

\[ \sim \left( \frac{m_\mu}{m_e} \right)^2 \sim 40,000 \]

Which is easy to understand from the uncertainty principle.
Theory for Muon \((g - 2)\)

\[
a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{hadronic}) + a_\mu(\text{weak})
\]

\[
a_\mu(\text{New Physics}) = a_\mu(\text{Measured}) - a_\mu(\text{SM})
\]
The Experimental Technique

Protons from AGS

Target

Pions

\[ p = 3.1 \text{ GeV/c} \]

\[ \pi^+ \rightarrow \mu^+ \nu_\mu \]

Inflector

Injection Orbit

Storage Ring

Ideal Orbit

Kicker Modules

\[ x_c \approx 77 \text{ mm} \]

\[ \beta \approx 10 \text{ mrad} \]

\[ B \cdot dl \approx 0.1 \text{ Tm} \]
Inflector Geometry
Inflector Exit Geometry

- Outer cryostat
- Upper pole: \( \rho = 7112 \text{ mm} \)
- Lower magnet pole
- Heat shield

Muon storage Region 45 mm R
Spin and Momentum Precession

\[ \omega_C = \frac{eB}{mc\gamma} \quad \omega_S = \frac{geB}{2mc} + (1 - \gamma)\frac{eB}{\gamma mc} \]

\[ \omega (\vec{S} \text{ relative to } \vec{p}) \quad \omega_a = \omega_S - \omega_C = \left(\frac{g - 2}{2}\right)\frac{eB}{mc} \]

The highest energy \( e^\pm \) carry the spin information
The need for vertical focusing

- In \((g - 2)\) we store for \(\sim 4000\) turns.
- Magnetic focusing conflicts with the need to know \(B\) to 0.1ppm.
- Can we use an electric field?

Spin Motion in \(\vec{E}\) and \(\vec{B}\)-fields:

\[
\tilde{\omega}_a = \frac{d\Theta_R}{dt} = \frac{e}{mc} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right]
\]

For \(\gamma = 29.3\)

\[
\left( a_\mu - \frac{1}{\gamma^2 - 1} \right) = 0
\]
The Ring Layout

- inflector
- Q4
- trolley drive
- traceback chambers
- FBM
- cryo pump
- FBM
- Q1
- Q2
- Q3
- K1
- K2
- K3

Quads cover 43% of the ring.
The Ring $\beta$–function

\[ \beta_y \]

\[ \beta_x \]

$\phi$ (degrees)

$\beta$ (meters)
The Kicker Plates
The Kicker Current Pulse

Kicker Current (kA)

- Beam

Time (ns)

- 200
- 400
- 600
- 800
- 1000

Power Supply ~1500 V

"charge"

300 µf

"charge now"

80M Ω

11.5Ω

10nf

CT

20M Ω

1000 pf

In air

In oil

In vacuum
A Kicker Modulator
Magnetic Circuits

\[ \Phi \oint \frac{d\ell}{\mu A} = NI \quad \Phi\mathcal{H} = MMF \quad \text{Ohm's law} \]
An array of 17 NMR probes on the trolley maps the B Field in the storage region.
Installation of a Pole Piece
The Nude Storage Ring
# Storage Ring Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>(g-2)</em> Frequency</td>
<td>( f_a \sim 0.23 \times 10^6 \text{ Hz} )</td>
<td>( \tau_a = 4.37 \mu s )</td>
</tr>
<tr>
<td>Muon kinematics</td>
<td>( p_\mu = 3.094 \text{ GeV/c} )</td>
<td>( \gamma_\mu = 29.3 )</td>
</tr>
<tr>
<td></td>
<td>( \gamma \tau = 64.4 \mu s )</td>
<td></td>
</tr>
<tr>
<td>Cyclotron Period</td>
<td>( \tau_{cyc} = 149 \text{ ns} )</td>
<td></td>
</tr>
<tr>
<td>Central Radius</td>
<td>( \rho = 7112 \text{ mm} )</td>
<td>( 280'' )</td>
</tr>
<tr>
<td>( B_0 = 1.451 \text{ T} )</td>
<td>Storage Aperture</td>
<td>9.0 cm circle</td>
</tr>
<tr>
<td>In one lifetime:</td>
<td>432 revolutions around ring</td>
<td></td>
</tr>
<tr>
<td></td>
<td>14.7 <em>(g-2)</em> periods</td>
<td></td>
</tr>
</tbody>
</table>
Mapping the $\vec{B}$ field.

NMR trolley, 17 probes to map the field.

366 fixed NMR probes monitor field stability.
$B(0, 0, \phi)$ in 1999
\[ \langle B \rangle = \int M(r, \theta) B(r, \theta) r \, dr \, d\theta \]

\[ B(r, \theta) = \sum_{n=0}^{\infty} r^n [c_n \cos n\theta + s_n \sin n\theta] \]

One slice in azimuth.

Muon Distribution

\[ M(r, \theta) = \sum [\gamma_m(r) \cos m\theta + \sigma_m(r) \sin m\theta] \]
And $\langle B \rangle_\phi$ is:

**Multipole expansion of B field in 1999**

### 1ppm field contours

<table>
<thead>
<tr>
<th>Multipole</th>
<th>Normal (ppm)</th>
<th>Skew (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quad</td>
<td>-2.23</td>
<td>2.17</td>
</tr>
<tr>
<td>Sext</td>
<td>-1.15</td>
<td>2.52</td>
</tr>
<tr>
<td>Octu</td>
<td>-1.33</td>
<td>1.85</td>
</tr>
<tr>
<td>Decu</td>
<td>0.92</td>
<td>0.95</td>
</tr>
</tbody>
</table>

**2000**

### 1ppm field contours

-1 - -1 - 0 - +1

B. Lee Roberts, EPAC02, 4 June 2002 – p.28/44
Experimental Signal is $\mu$ Decay

The Muon Rest Frame

Highest energy $e^+$ are along muon spin
The electron carries the muon spin
The $e^\pm$ Energy Spectrum

\[ \delta \varepsilon = \frac{\delta \omega_a}{\omega_a} = \frac{\sqrt{2}}{2\pi f_a \tau \mu N^{1/2} A} \]

\[ \int N A^2 \equiv \int_{E_T}^{E_{\text{max}}} N(E) A^2(E) dE \]
The Detector Geometry

- Muon momentum
- Muon spin
- Sci-Fi Calorimeter module
- Measures Energy and time
- Spin forward, more high energy e
- Spin backward, less high energy e
- 400 MHz digitizer
\[
f(t) = N_0 e^{-\lambda t} \left[ 1 + A \cos(\omega_a t + \phi) \right]
\]
The Muon Distribution

$e^+$ Time Spectrum: $t = 6 \ \mu s$

$e^+$ Time Spectrum: $t = 36 \ \mu s$

The distribution of equilibrium radii

- data
- simulation
\( \omega_a \textbf{Fitting Function} \)

Nature gives us 5 parameters:

\[
f(t) = N_0 e^{-\lambda t} [1 + A \cos(\omega_a t + \phi)]
\]

Storage ring plus bunched beam gives more:

Fourier Transform of the residuals from a 5–parameter fit (from 1 detector).
Weak Focussing \[ n = \frac{\kappa R_0}{\beta B_0} \]

\( \kappa = \text{electric quadrupole gradient}; \quad n \simeq 0.137 \)

\[ \gamma m \ddot{x} + \frac{\gamma mv^2}{R_0^2} (1 - n) x = 0; \quad \gamma m \ddot{y} + k e y = 0 \]

\[ f_y = f_C \sqrt{n} \simeq 0.37 f_C; \quad f_x = f_C \sqrt{1 - n} \simeq 0.929 f_C \]

Detector acceptance depends on \( r \). The beam moves radially relative to one detector with the “Coherent Betatron Frequency”

\[ f_{CBO} = f_C - f_x = (1 - \sqrt{1 - n}) f_C \]

which amplitude modulates the \( e^{\pm} \) signal.
The Tune Plane

- $2n - 2n = 1$
- $3n - 2n = 2$
- $n - 2n = 0$
- $n + n = 1$
- $3n + n = 3$
- $4n + n = 4$
- $n = 1$
- $5n = 2$
- $3n = 1$
- $n - 3n = 0$
- $2n - 3n = 1$

Points:
- $n = 0.142$
- $n = 0.137$
- $n = 0.122$
Frequencies in the \((g - 2)\) ring.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Expression</th>
<th>Frequency</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_a)</td>
<td>(\frac{e}{2\pi mc} a_{\mu} B)</td>
<td>0.23 MHz</td>
<td>4.37 (\mu)s</td>
</tr>
<tr>
<td>(f_c)</td>
<td>(\frac{v}{2\pi R_0})</td>
<td>6.7 MHz</td>
<td>149 ns</td>
</tr>
<tr>
<td>(f_x)</td>
<td>(\sqrt{1 - n f_c})</td>
<td>6.23 MHz</td>
<td>160 ns</td>
</tr>
<tr>
<td>(f_y)</td>
<td>(\sqrt{n f_c})</td>
<td>2.48 MHz</td>
<td>402 ns</td>
</tr>
<tr>
<td>(f_{\text{CBO}})</td>
<td>(f_c - f_x)</td>
<td>0.477 MHz</td>
<td>2.10 (\mu)s</td>
</tr>
<tr>
<td>(f_{\text{VW}})</td>
<td>(f_c - 2f_y)</td>
<td>1.74 MHz</td>
<td>0.574 (\mu)s</td>
</tr>
</tbody>
</table>
Fiber Beam Monitors

x monitor

calibrate

y monitor

calibrate
Measuring the Tune with FBM

![Graphs showing x and y fiber data](image)

Harp 1 (Horizontal), Fiber 4 ns

Harp 2 (Vertical), Fiber 4 ns
Beam Centroid and Scraping

- **cbo – fiber harps**
  - No Scraping, \( f_\beta = 472 \) kHz
  - 7 kV Scraping, \( f_\beta = 416 \) kHz

- **FFT – fiber harps**
  - Muon Fast Rotation Frequency
  - Proton Fast Rotation Frequency
  - \( f_c(1 - 2\sqrt{n}) \)
  - \( f_c(1 - \sqrt{n}) \)

- **cbo – traceback**
  - Fraction of CBO Period
  - Range (cm)
\[ \omega_a = \frac{e}{m} a_\mu < B > \]

Remove offsets and divide to determine

\[ R = \frac{\omega_a}{\omega_p} \]

From

\[ a_\mu = \frac{R}{\lambda - R} \quad \lambda = \frac{\mu_\mu}{\mu_p} \]

Add corrections for radial \( \vec{E} \)-field and vertical “pitching motion”. (\( \sim 0.81 \pm 0.08 \text{ ppm} \))
Agreement with SM?

Experiment - Theory = + 1.6 \sigma
Agreement with SM?

requiring that 2000 answer agree with ’99 result to ±2σ gives the range of values — SM agreement

−1σ to +5.5σ

\( a_\mu \)
Conclusions and Outlook

- The storage ring, kicker, quadrupoles, all meet their specifications in the \((g - 2)\) experiment.
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- Beam dynamics is quite important in understanding the data.
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- Beam dynamics is quite important in understanding the data.
- Two data sets are being analyzed:
  \(\sim \pm 0.77 \text{ ppm } \mu^+ \text{ from 2000 run.}\)
  \(\sim \pm 0.8 \text{ ppm } \mu^- \text{ from 2001 run.}\)
Conclusions and Outlook, ctd.

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Stay tuned!