Solutions to PY231 Test 2, Spring 2003

1) (10 pts) In her notes describing her new composition *The Light of the End*, which was given its world premiere last weekend at the Boston Symphony, Sofia Gubaidulina says:

“The conflict in this piece arises between a theme consisting exclusively of sounds from the natural overtones and a theme that uses the 12-tone tempered scale.

The duet of French horn and cello before the central expressive tutti sounds especially antagonistic: the horn and cello play one and the same melody in different tunings, the natural and tempered. Great dissonance.”

Explain what the composer is talking about. Assuming a harmonic series based on C₂, tell which notes will be very out of tune between the two, which will be a little out of tune, and which will be more or less ok. Will any of them be in tune?

**When playing in “unison” but with different tunings, equal temperment and the members of the harmonic series will not agree on many of the tunings. The octaves will be in tune (the Cs in the harmonic series), the G, D will be ok, the E will be off, and the 7th, 11, 13th and 14 th harmonics will be way out of tune with each other.**

2a) (20 pts 10, 10) Calculate the frequencies for the just scale, which makes the compromise that the tonic, sub dominant and dominant major triads are beat free. For the key of C, this means that the C, F and G major triads are beat free. You should assume that the frequency for the lower C is 1, and for the upper one, it is 2.

2b) Now find the interval between each note (a frequency ratio). What intervals do you find in this scale for the M2?

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<table>
<thead>
<tr>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>freq</td>
<td>1/8</td>
<td>5/4</td>
<td>4/3</td>
<td>3/2</td>
<td>5/3</td>
<td>15/8</td>
<td>2/1</td>
</tr>
<tr>
<td>interv</td>
<td>9/8</td>
<td>10/9</td>
<td>16/15</td>
<td>9/8</td>
<td>10/9</td>
<td>9/8</td>
<td>16/15</td>
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</tbody>
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It is necessary to calculate the frequencies using these triads, C-E-G, F-A-C and G-B-D. I illustrate a few of these.

**C-E-G:** Since C has the frequency 1, the E and G come from the harmonic series, so $E = 5/4$ and $G = 3/2$.

Since G is 3/2 and the triad G-B-D has to be also of the ratios of the harmonic series, we have

$$f_B = f_G \times \text{frequency ratio for a M3} = f_G \times \frac{5}{4} = \frac{15}{8}$$

Also

$$f_D = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$$
which when taken down an octave is 9/8.

Now for the F-A-C triad, F can be determined by going down a P5 from from the upper C, so

\[ f_F = 2 \times \frac{2}{3} = \frac{4}{3} \]

and

\[ f_A = f_F \times \frac{5}{4} = \frac{4}{3} \times \frac{5}{4} = \frac{5}{3} \]

To find the interval between adjacent notes, we divide the upper frequency by the lower one. For example, to find the interval between F and G, we have

\[ \frac{3}{2} \times \frac{3}{4} = \frac{9}{8} \]

3) (22 pts, 2 pts per answer) Sketch the first 4 modes of a string fixed at both ends. How are the frequencies related to each other for a uniform perfectly flexible string? They are harmonically related, with all harmonics present.

3b) Sketch the first 2 modes of a a stopped pipe and an open pipe. Please sketch the pressure, not displacement.

3c) The modes of the air columns in these two pipes are harmonically related, if we ignore the end effects. Which of these two pipes has mode frequencies which correspond to all harmonics, and which one has modes which are only the odd harmonics? The stopped pipe has only odd harmonics, and the open pipe has all harmonics.

3c) If these two pipes were in a pipe organ and had the same length, which would be lower in pitch and by how much would it be lower? The stopped pipe would be an octave below the open one.
(4) (22 pts 2 pts each)

4a) Sketch the first five modes of a drum head.

4b) Sketch the first three modes of a free bar, and the lowest torsional mode.

4c) Sketch the first two modes of a rod clamped at one end.
5) (16 pts, 2 pts each) Using the figures on the previous two problems, to answer the following questions. (for the drum and bar questions sketch the striking and damping points on the figure provided.)

5a) Where would you pluck a string to excite only the odd modes? in the middle where the even modes have a node
5b) Which modes would be missing if you plucked one quarter of the way along the string? If you pluck \(1/4\) of the way along the string, this is the location of nodes of the 4th, 8th, 12th, and any multiple of 4 mode. They will be absent.

5c) Where would you pluck to get a large amount of the higher modes? near the end of the string.

5d) What would the pitch jump by if you plucked the string, and then touched it in the middle? If you touch it in the middle, you will disrupt modes 1, 3, ... (all odd) and thus the pitch will jump an octave. (the even modes have a node at the middle and will not be affected).

5e) What would the pitch jump by if you plucked it and then touched it \(1/3\) of the way along the string. \(1/3\) of the way along the string, the 3rd, 6th, ... (multiples of 3) modes will have nodes and be unaffected. All the others will be damped, so the pitch will jump from \(f\) to \(3f\) or a P12.

5f) Show where would you strike a drum head to maximally excite the first mode. Which modes will be eliminated by this choice of striking point. In the middle. All modes with a node in the middle would be eliminated by this striking point.

![strike here]

5g) Tell how you would strike and damp the drum to excite only mode 2. You would strike about \(1/4\) of the way across the head, and use your hand to force a nodal line through a diameter of the drum head.

![strike here and damp here]

5h) Where would you strike the bar to maximally excite the torsional mode? on any corner
6) (10 pts) Impedance plays an important role in the establishment of a standing wave. The bowed violin is an example of such a system. (a) Explain how the standing wave is set up in the violin string. (b) A plucked violin string decays rather quickly compared to a guitar or piano string. In terms of energy, and the way that each instrument is played explain why this is reasonable. (c) Now explain why a violin which had an extremely short decay time when it was plucked (say 100 or 1000 times faster than a normal violin) might be difficult to play well.

(a) When a wave comes to a boundary, the difference in the characteristic impedance between the two media determines how much of the wave is reflected and how much is transmitted. If the impedance mismatch is large, most of the energy of the wave will be reflected. In the violin when the string breaks free from the bow, the kink which was created by pulling the string aside travels down the string, and is mostly reflected from the bridge. The superposition of two identical waves travelling in opposite directions will set up a standing wave (if they are of the correct wavelength). Since the shape of the string just before it breaks free can be described as a superposition of the normal modes of a string, a number of modes participate in the vibration, and the waves are of the correct wavelength.

(b) Since the violin is usually bowed, we want enough of the wave to be reflected to set up a stable standing wave and regime of oscillation, and we want a substantial amount of energy to make its way to the body of the instrument to be radiated into the room as sound. The piano and guitar are impulsively excited, and thus if too much of the energy goes into the bridge at once, the sound will decay rapidly, and it will interfere with the ability to play melodic passages. Thus they have evolved such that they decay (ring) slower (longer) than the strings on a violin when they are struck or plucked.

(c) If the decay time is much shorter, then the impedance mismatch would be much less. Thus very little of the wave would be reflected back, and it would be difficult to set up standing waves. Thus the feedback needed for a stable regime of oscillation would be missing and it would not play very easily. Starting a note would be particularly difficult.