Recent News from the Muon \((g - 2)\) Experiment at BNL

The \(\pm 0.7\) ppm measurement

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Boston University
BNL AGS E821:

A New Precision Measurement of the Muon \((g - 2)\) Value at the level of 0.35 ppm

Boston University, Brookhaven National Laboratory, Budker Institute of Nuclear Physics - Novosibirsk, Cornell University, KEK, KVI and Rijksuniversiteit - Groningen, University of Heidelberg, University of Illinois, University of Minnesota, Tokyo Institute of Technology, Yale University
Present E821 Collaboration

R.M. Carey, X. Huang, A. Lam-Ng, I. Logashenko, J.P. Miller, J. Paley, B.L. Roberts,†
Outline of the Talk

- Brief Introduction to \((g - 2)\)
- Overview of the Experimental Technique
- Beam Dynamics in the \((g - 2)\) Storage Ring
- Data Analysis
- Summary and Conclusions
Muon: (2\textsuperscript{nd} generation lepton)

\[ m_\mu c^2 = 105.658\,389(34) \text{ MeV} \]
\[ \tau_\mu = 2.197\,03(4) \mu\text{s} \]

Source: \( \pi^- \rightarrow \mu^- \bar{\nu}_\mu \quad \text{Weak Decay} \)

Parity Violating Decay \( \Rightarrow \) Polarized Muons

Weak Decay: \( \mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \)
Magnetic Moments, $g$-Factors, etc.

\[ \vec{\mu}_s = g_s \left( \frac{e}{2m} \right) \vec{s} \]

$\vec{\mu}$ - magnetic moment; $g$ - gyromagnetic ratio
\( \vec{s} \) is the spin.

- Dirac Equation Predicts $g \equiv 2$
- In nature radiative corrections make $g \neq 2$.

\[ g = 2 + \frac{\alpha}{\pi} + \cdots \]

Dirac

Kusch and Foley, Schwinger, 1947
Magnetic Moments: Definitions and Values

\[ \mu = \left( 1 + a \right) \frac{e\hbar}{2m} \quad \text{where} \quad a = \left( \frac{g - 2}{2} \right) \]

\[ \mu_e = 1.001 \, 159 \, 652 \, 193 \, \frac{e\hbar}{2m_e} ; \quad \text{For comparison:} \]

\[ \mu_\mu = 1.001 \, 165 \, 923 \, \frac{e\hbar}{2m_\mu} ; \quad \mu_p = 2.792 \, 847 \, 39 \, \frac{e\hbar}{2m_p} \]

\[ g_p = 5.5857 \cdots \neq 2 \]
**Theory for Muon \((g - 2)\)**

(This has been covered by A. Czarnecki)

\[
a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{hadronic}) + a_\mu(\text{weak})
\]

\[
a_\mu(\text{New Physics}) = a_\mu(\text{Measured}) - a_\mu(\text{SM})
\]
The Experimental Technique

Protons from AGS → Pions → $\pi^+ \rightarrow \mu^+ \nu_\mu$

$\chi_c \approx 77$ mm
$\beta \approx 10$ mrad
$B \cdot dl \approx 0.1$ Tm

B. Lee Roberts, τ02, UCSD, 11 September 2002 – p.9/4
Inflector Geometry
Inflector Exit Geometry

Incident $\mu$ phase space not matched to the ring.
Spin and Momentum Precession

\[
\omega_C = \frac{eB}{mc\gamma} \quad \omega_S = \frac{geB}{2mc} + (1 - \gamma)\frac{eB}{\gamma mc}
\]

\[
\omega (\vec{S} \text{ relative to } \vec{p}) \quad \omega_a = \omega_S - \omega_C = \left(\frac{g - 2}{2}\right)\frac{eB}{mc}
\]

Spin Motion in $\vec{E}$ and $\vec{B}$ Fields.

\[
\vec{\omega}_a = \frac{d\Theta_R}{dt} = \frac{e}{mc} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right]
\]

for $\gamma = 29.3 \quad \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) = 0$
In a uniform $\vec{B}$ field all muons precess at the same rate.
Schematic of the Magnet

An array of 17 NMR probes on the trolley maps the B Field in the storage region

ρ = 7112 mm
Installation of a Pole Piece
### Storage Ring Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g-2) Frequency</td>
<td>( f_a \sim 0.23 \times 10^6 \text{ Hz} )</td>
<td>( \tau_a = 4.37 \mu \text{s} )</td>
</tr>
<tr>
<td>Muon kinematics</td>
<td>( p_\mu = 3.094 \text{ GeV/c} )</td>
<td>( \gamma_\mu = 29.3 )</td>
</tr>
<tr>
<td></td>
<td>( \gamma \tau = 64.4 \mu \text{s} )</td>
<td></td>
</tr>
<tr>
<td>Cyclotron Period</td>
<td>( \tau_{\text{cyc}} = 149 \text{ ns} )</td>
<td></td>
</tr>
<tr>
<td>Central Radius</td>
<td>( \rho = 7112 \text{ mm} )</td>
<td>(280&quot;)</td>
</tr>
<tr>
<td>( B_0 = 1.451 \text{ T} )</td>
<td>Storage Aperture</td>
<td>9.0 cm circle</td>
</tr>
<tr>
<td>In one lifetime:</td>
<td>432 revolutions around ring</td>
<td></td>
</tr>
<tr>
<td></td>
<td>14.7 (g-2) periods</td>
<td></td>
</tr>
</tbody>
</table>
NMR Trolley with 17 Probes

\[ < B > = < \int M(r, \theta) B(r, \theta) r dr d\theta >_\phi \]

366 fixed NMR probes monitor field stability.
And $< B >_\phi$ is:

In 1999 Quadrupole $\approx 2.0$ ppm of $B_0$, in 2000 $\approx 0.2$ ppm of $B_0$. 
**Systematic Errors on** $<\omega_p>$

From two independent analyses of $<\omega_p>$.

<table>
<thead>
<tr>
<th>Source</th>
<th>1999 (ppm)</th>
<th>2000 (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflector Fringe Field</td>
<td>0.20</td>
<td>-</td>
</tr>
<tr>
<td>Calibration of trolley probes</td>
<td>0.20</td>
<td>0.15</td>
</tr>
<tr>
<td>Interpolation with fixed probes</td>
<td>0.15</td>
<td>0.10</td>
</tr>
<tr>
<td>Trolley measurements of $B_0$</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Uncertainty from $\mu$-distribution</td>
<td>0.12</td>
<td>0.03</td>
</tr>
<tr>
<td>Absolute calibration</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Others†</td>
<td>0.15</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Total systematic error on</strong> $\omega_p$</td>
<td><strong>0.4</strong></td>
<td><strong>0.24</strong></td>
</tr>
</tbody>
</table>

†Higher multipoles, trolley temperature stability, kicker eddy currents.
Experimental Signal is $\mu$ Decay

The Muon Rest Frame

Highest energy $e^+$ are along muon spin
The electron carries the muon spin
The $e^{\pm}$ Energy Spectrum

\[ \delta \varepsilon = \frac{\delta \omega_a}{\omega_a} = \frac{\sqrt{2}}{2\pi \int_a \tau \mu N^{\frac{1}{2}} A} \]

\[ \int N A^2 \equiv \int_{E_T}^{E_{max}} N(E) A^2(E) \, dE \]
**The Detector Geometry**

- **muon momentum**
- **muon spin**
- Sci-Fi Calorimeter module
  - Measures Energy and time
- **spin forward, more high energy e**
- **spin backward, less high energy e**
- **400 MHz digitizer**
Time Spectrum, $E > 2.0 \text{GeV}$

$\sigma_{\text{stat}} \approx 0.7 \text{ ppm}$

$$f(t) = N_0 e^{-\lambda t} [1 + A \cos(\omega_a t + \phi)]$$
The Muon Distribution

The distribution of equilibrium radii

\[ <R> = 711.436 \text{ cm}, \text{ RMS Width} = 0.955 \]
\[ <R> = 711.460 \text{ cm}, \text{ RMS Width} = 0.999 \]

Magic Radius 711.2 cm
Peak 711.093 cm

\[ e^+\text{ Time Spectrum: } t = 6 \mu s \]
\[ e^+\text{ Time Spectrum: } t = 36 \mu s \]

Measured distribution and Monte Carlo agree.
Fitting Function

Nature gives us 5 parameters:

\[ f(t) = N_0 e^{-\lambda t} \left[ 1 + A \cos(\omega_\alpha t + \phi) \right] \]

Storage ring plus bunched beam gives more:

Fourier Transform of the residuals from a 5–parameter fit (from 1 detector).
**Weak Focussing** \[ n = \frac{\kappa R_0}{\beta B_0} \]

\( \kappa = \) electric quadrupole gradient; \( n \approx 0.137 \)

\[ \gamma m \ddot{x} + \frac{\gamma m v^2}{R_0^2} (1 - n) x = 0; \quad \gamma m \ddot{y} + \kappa e y = 0 \]

\[ f_y = f_C \sqrt{n} \approx 0.37 f_C; \quad f_x = f_C \sqrt{1 - n} \approx 0.929 f_C \]

Detector acceptance depends on \( r \). The beam moves radially relative to one detector with the “Coherent Betatron Frequency”

\[ f_{CBO} = f_C - f_x = (1 - \sqrt{1 - n}) f_C \]

which amplitude modulates the \( e^\pm \) signal.
Frequencies in the \((g - 2)\) ring.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Expression</th>
<th>Frequency</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_a)</td>
<td>(\frac{e}{2\pi mc}a_\mu B)</td>
<td>0.23 MHz</td>
<td>4.37 (\mu)s</td>
</tr>
<tr>
<td>(f_c)</td>
<td>(\frac{v}{2\pi R_0})</td>
<td>6.7 MHz</td>
<td>149 ns</td>
</tr>
<tr>
<td>(f_x)</td>
<td>(\sqrt{1 - nf_c})</td>
<td>6.23 MHz</td>
<td>160 ns</td>
</tr>
<tr>
<td>(f_y)</td>
<td>(\sqrt{nf_c})</td>
<td>2.48 MHz</td>
<td>402 ns</td>
</tr>
<tr>
<td>(f_{CBO})</td>
<td>(f_c - f_x)</td>
<td>0.477 MHz</td>
<td>2.10 (\mu)s</td>
</tr>
<tr>
<td>(f_{VW})</td>
<td>(f_c - 2f_y)</td>
<td>1.74 MHz</td>
<td>0.574 (\mu)s</td>
</tr>
</tbody>
</table>
The Tune Plane

\[ \nu_x = \sqrt{1 - n} \]
\[ \nu_y = \sqrt{n} \]

Resonance Condition:

\[ J \nu_x + K \nu_y = L \]

\( J, K, L \) integer

\[ 2\nu_y = 1 \]
\[ \nu_y - 4\nu_y = -1 \]
\[ \nu_y - 2\nu_y = 0 \]
\[ 2\nu_y - 3\nu_y = 1 \]
\[ n = 0.137 \]
\[ n = 0.122 \]

\[ 3\nu_y = 1 \]
\[ \nu_x - 3\nu_y = 0 \]
\[ 3\nu_x + \nu_x = 1 \]
\[ n = 0.142 \]
\[ n = 0.137 \]

\[ 5\nu_y = 2 \]
\[ \nu_x = 3\nu_y \]
\[ \nu_x = 2\nu_y \]
\[ \nu_x = 1 \]

\[ 4\nu_y = 2 \]
\[ \nu_x = 4\nu_y \]
\[ \nu_x = 2\nu_y \]
\[ \nu_x = 1 \]

\[ B. \text{Lee Roberts}, \tau_0^2, \text{UCSD, 11 September 2002 – p.29/41} \]
Effect of CBO on Fit to Muon Decay Spectrum

- CBO causes modulation of N, amplitude $\approx 0.01$
- CBO causes modulation of observed energy distribution,
- which in turn causes oscillation in $A(E)$, $\phi(E)$, with amplitudes $\approx 0.001$, $\approx 1$ mrad.
Modified Time Distribution

\[ N_p = N_0 e^{\frac{-t}{\tau}} (1 + A' \sin (\omega_a t + \phi')) \times (1 + A_{CB0}(t) \cos (\omega_{CB0} t + \phi_{CB0})) \]

- \( A' = A (1 + A_1(t) \cos (\omega_{CB0} t + \phi_1)) \)
- \( \phi' = \phi (1 + A_2 \cos (\omega_{CB0} t + \phi_2)) \)
- \( A_1 \) and \( A_2 \) → artificial shifts in \( \omega_a \) up to 4 ppm in individual detectors when not accounted for.

Shifts largely cancel in sum of detectors due to circular symmetry. Cancellation factor \( \approx 9 \)
Fitting without and with CBO

- 5 par fit + PU + $\mu$-loss:
  \[ f_a = (229.07064 \pm 0.00014) \text{ kHz}; \quad \chi^2/\nu = 24.3/19 \]
  \[ A_{\text{sine}} = (1.2 \pm 0.20) \times 10^{-3} \text{ kHz}, \]

- full fit + PU + $\mu$-loss:
  \[ f_a = (229.07054 \pm 0.00016) \text{ kHz}; \quad \chi^2/\nu = 23.9/21 \]
Pulse Pileup

pileup changes sign

pileup

49.24 μs - 66.70 μs

all positrons

(scaled by 3.08)
Muon Losses

Muon Losses vs. Time

Proton correction
Four Separate Analyses for $\omega_a$

- 1. fits $E > 1.4$ GeV with 0.2 GeV energy bins
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- 2. multiparameter fits with one energy bin, $E > 2.0$ GeV
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- 2. multiparameter fits with one energy bin, $E > 2.0$ GeV
- 3. similar to 2 with different PU treatment, complementary systematic studies.
- 4. “ratio” fit which divides out slowly varying effects.
Results for $\omega_a$

$$\omega_a = 2\pi f_0 (1 - R \times 10^{-6}), \quad f_0 = 229.1 \text{ kHz}$$

<table>
<thead>
<tr>
<th>Fit</th>
<th>R(ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>128.71(.70)(.28)</td>
</tr>
<tr>
<td>2</td>
<td>128.57(.64)(.37)</td>
</tr>
<tr>
<td>3</td>
<td>128.31(.64)(.36)</td>
</tr>
<tr>
<td>4</td>
<td>128.48(.63)(.34)</td>
</tr>
<tr>
<td>Mean</td>
<td>128.518(.615)(.309)</td>
</tr>
</tbody>
</table>

Expected range of values 0.5 ppm
**Systematic Errors (ppm) on \( \omega_a \)**

<table>
<thead>
<tr>
<th>Source</th>
<th>1999</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pile-Up</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>AGS Background</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>Lost Muons</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Timing Shifts</td>
<td>0.10</td>
<td>0.02</td>
</tr>
<tr>
<td>E-field and vertical ( \beta )-motion</td>
<td>0.08</td>
<td>0.03</td>
</tr>
<tr>
<td>Fitting Method / Binning</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>Coherent Betatron Oscillation</td>
<td>0.05</td>
<td>0.21</td>
</tr>
<tr>
<td>Beam debunching</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Detector Gain Changes</td>
<td>0.02</td>
<td>0.13</td>
</tr>
<tr>
<td><strong>Total Systematic on ( \omega_a )</strong></td>
<td><strong>0.3</strong></td>
<td><strong>0.31</strong></td>
</tr>
</tbody>
</table>
\[ \omega_a = \frac{e}{m} a_\mu < B > \]

Remove offsets and divide to determine

\[ R = \frac{\omega_a}{\omega_p} \]

From

\[ a_\mu = \frac{R}{\lambda - R} \]

\[ \lambda = \frac{\mu_\mu}{\mu_p} \]

Add corrections for radial \( \vec{E} \)-field and vertical “pitching motion”. (+0.81 ± 0.08 ppm)

\[ a_{\mu+} = 11659204(7)(5) \times 10^{-10} \ (±0.7 \text{ ppm}) \]
Measurements of $a_\mu$

Theory (DH98)

$\frac{116 590 000}{116 591 000}$ $\frac{116 592 000}{116 593 000}$ $\frac{116 594 000}{116 595 000}$ $10^{-11}$

(10 ppm) (9.4 ppm) CERN $\mu^+$

(13 ppm) E821 (97) $\mu^+$

(5 ppm) E821 (98) $\mu^+$

(1.3 ppm) E821 (99) $\mu^+$

(0.7 ppm) E821 (00) $\mu^+$
Measurements of $a_\mu$

Brookhaven Muon g-2 Data

<table>
<thead>
<tr>
<th>Year</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNL'98</td>
<td>$a_\mu \times 10^{10} = 11659000$</td>
</tr>
<tr>
<td>BNL'99</td>
<td>$a_\mu \times 10^{10} = 180$</td>
</tr>
<tr>
<td>BNL'00</td>
<td>$a_\mu \times 10^{10} = 200$</td>
</tr>
<tr>
<td>World Average</td>
<td>$a_\mu \times 10^{10} = 220$</td>
</tr>
</tbody>
</table>

Standard Model Prediction

- DEHZ $\tau$
- DEHZ $ee$
- DH'98

Status of the Experiment

- The storage ring, kicker, quadrupoles, all meet their specifications in the \((g - 2)\) experiment.

Beam dynamics is quite important in understanding the data. One additional data set is still being analyzed: ppm from 2001 run.
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- Beam dynamics is quite important in understanding the data.
- One additional data set is still being analyzed: \(\sim \pm 0.8 \text{ ppm } \mu^-\) from 2001 run.
- The President’s Budget recommended termination of HEP at the AGS....
  \(\Rightarrow\) No more data collection???
Conclusions and Outlook

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- This is clearly a work in progress! Both theory and experiment will be further refined in the future.
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Stay tuned!