Muon \((g - 2)\): The Latest Word

83 Years of Magnetic Moments

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Outline of the Talk

- Introduction to \((g - 2)\)
- Motivation and Theory
- Experimental Details
- Data Analysis
  - \(\omega_p\)
  - \(\omega_a\)
- Results
- An Intermezzo: “\(\tau\) for two”
- Summary and Outlook
Fundamental Questions in Particle Physics

Why Matter?

CP Violation

Why Mass?

The Higgs Field

What's Beyond the Standard Model

Supersymmetry or other extensions?
Fundamental Questions in Particle Physics

- Why Matter?
  - CP - Violation

What's Beyond the Standard Model
Supersymmetry or other extensions?
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- Why Matter?
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Fundamental Questions in Particle Physics

- Why Matter?
  \( CP \) - Violation

- Why Mass?
  The Higgs Field

- What’s Beyond the Standard Model
  Supersymmetry or other extensions?
Paths to the Frontier
Paths to the Frontier

- The Highest Energy
Paths to the Frontier

- The Highest Energy
- The Highest Precision
Paths to the Frontier

- The Highest Energy
- The Highest Precision

Today we will focus on the Precision Path.
The Standard Model

Leptons

\[
\begin{array}{ccc}
e & \mu & \tau \\
\nu_e & \nu_\mu & \nu_\tau \\
\end{array}
\]

interact weakly through the EW gauge bosons

Quarks

\[
\begin{array}{ccc}
u & c & t \\
d & s & b \\
\end{array}
\]

interact strongly through the gluon \( g \)

Electroweak Gauge Bosons

\[
\begin{array}{ccc}
\gamma & Z^0 & W^\pm \\
\end{array}
\]
The Muon

"Who ordered that?" I.I. Rabi

\[ m_\mu c^2 = 105.658\,389(34) \text{ MeV} \]
\[ \tau_\mu = 2.197\,03(4) \mu s \]

Usual Source:

Weak Decay: \( \pi^- \rightarrow \mu^- \bar{\nu}_\mu \)

Parity Violating Decay \( \Rightarrow \) Polarized Muons

Weak Decay: \( \mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \)
Ein Weg zur experimentellen Prüfung der Richtungsquantelung im Magnetfeld.

Von Otto Stern in Frankfurt a. M.

Mit zwei Abbildungen. — (Eingegangen am 26. August 1921.)

In der Quantentheorie des Magnetismus und des Zeemaneffektes wird angenommen, daß der Vektor des Impulsmomentes eines Atoms nur ganz bestimmte diskrete Winkel mit der Richtung der magnetischen Feldstärke Σ bilden kann, derart, daß die Komponente des Impulsmomentes in Richtung von Σ ein ganzzahliges Vielfaches von $\hbar/2\pi$ ist. Bringen wir also ein Gas aus Atomen, bei denen das gesamte Impulsmoment pro Atom — die vektorielle Summe der Impulsmomente sämtlicher Elektronen des Atoms — den Betrag $\hbar/2\pi$ hat, in ein Magnetfeld, so sind nach dieser Theorie für jedes Atom nur zwei diskrete Lagen möglich, da die Komponente des Impulsmomentes in Richtung von Σ nur die beiden Werte $\pm \hbar/2\pi$ annehmen kann. Denken wir z. B. an einquantige Wasserstoffatome, so müssen die Ebenen der Elektronenbahnen sämtlich senkrecht auf Σ stehen.
By 1924 the famous “Stern-Gerlach” experiments were done, 3 papers in Z. Phys. and a review:
**ANNALEN DER PHYSIK.**

**VIERTE FOLGE. BAND 74.**

1. Über die Richtungsquantelung im Magnetfeld

von Walther Gerlach und Otto Stern.

(Neue Tafel III.)


Abfolgen ist. Die Ausmessung und Berechnung von den Aufnahmen Nr. 14 und Nr. 15 führte zu den Werten der folgenden Tabelle:

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Entfernung des unabgeleuchteten Strahles von der Scheibe</th>
<th>Mittlere Ablenkung des abgestoßenen Strahles</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>0,33 mm</td>
<td>berechnet 0,10 mm</td>
</tr>
<tr>
<td>14</td>
<td>0,21 mm</td>
<td>berechnet 0,14 mm</td>
</tr>
</tbody>
</table>

Die Genauigkeit der Messungen schätzen wir auf 10 Proz. Inhalt dieser Schreibgeschichten zeigen also die Versuche, daß das Silberatom im Normalzustand ein Bohrsches Magnetons hat.
Magnetic Moments, $g$-Factors, etc.

$$\vec{\mu}_s = g_s \left( \frac{e}{2m} \right) \vec{s}$$

If $\mu_e = 1$ Bohr magneton, $g_s = 2$.

- However Stern and Gerlach had compensating factors of 2 which made up for their ignorance of spin.
Magnetic Moments, \( g \)-Factors, etc.

\[
\vec{\mu}_s = g_s \left( \frac{e}{2m} \right) \vec{s}
\]

If \( \mu_e = 1 \) Bohr magneton, \( g_s = 2 \).

- However Stern and Gerlach had compensating factors of 2 which made up for their ignorance of spin.
- Uhlenbeck and Goudsmit had to invent spin for this to make sense.
Dirac Equation Predicts $g \equiv 2$

- For a NR $e^-$ in a weak $\vec{B}$-field:

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ \frac{p^2}{2m} - \frac{e}{2m} (\vec{L} + 2\vec{S}) \cdot \vec{B} \right] \psi$$

We see that Dirac $\Rightarrow g \equiv 2$, but in nature
**Dirac Equation Predicts** $g \equiv 2$

- For a NR $e^-$ in a weak $\vec{B}$-field:

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ \frac{p^2}{2m} - \frac{e}{2m} (\vec{L} + 2\vec{S}) \cdot \vec{B} \right] \psi$$

We see that Dirac $\Rightarrow g \equiv 2$, but in nature

- radiative corrections make $g \neq 2$. 

\[ g = 2 + \frac{\alpha}{\pi} \mu^* + c_2 \left( \frac{\alpha}{\pi} \right)^2 \mu^* + \epsilon^+ \]  

Dirac Kusch and Foley, Schwinger, 1947
Magnetic Moments: Definitions and Values

\[ \mu = (1 + a) \frac{e\hbar}{2m} \quad \text{where} \quad a = \left( \frac{g - 2}{2} \right) \]

\[ \mu_e = 1.001\,159\,652\,193 \frac{e\hbar}{2m_e} \]

\[ \mu_\mu = 1.001\,165\,923 \frac{e\hbar}{2m_\mu} \]
Magnetic Moments: Definitions and Values

\[ \mu = (1 + a) \frac{e\hbar}{2m} \]

where

\[ a = \left( \frac{g - 2}{2} \right) \]

\[ \mu_e = 1.001\,159\,652\,193 \frac{e\hbar}{2m_e}; \]

For comparison:

\[ \mu_p = 2.792\,847\,39 \frac{e\hbar}{2m_p} \]

\[ g_p = 5.5857 \cdots \neq 2 \]

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Magnetic Moment in a $\vec{B}$-Field

For an electron with $q = -e$, $AM = \vec{L}$:

$$\vec{\mu} = -g_L \frac{e\hbar}{2m} \vec{L}$$

- Energy: $E = -\vec{\mu} \cdot \vec{B}$
- Torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad \text{(Newton)}$$

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Larmor Precession (orbital AM)

An $e^-$ with magnetic moment $\vec{\mu}$ in a $\vec{B}$ field.

Larmor Frequency

$$\vec{\omega}_L = g_\ell \frac{\mu}{\hbar} \vec{B}$$

Experimentally $g_\ell = 1$
Other Magnetic Moment Effects

Atomic fine-structure splitting:

\[ j = l + s \]

\[ j = l - s \]

\[ \Delta E_{n,l} = \left( \frac{g_0}{2} + g_1 \right) \frac{(Z\alpha)^4}{2n^3} \frac{m}{\ell(\ell + 1)} \]

where: \( g = g_0 + g_1 \)
**Other Magnetic Moment Effects, ctd.**

Hyperfine Structure of Muonium \((\mu^+e^-)\)

Breit-Rabi Diagram for \(1^2S_{1/2}\) Ground State
Electric and Magnetic Dipole Moments for the Muon

\[ \mathcal{H} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E} \]

\( \vec{\mu} \cdot \vec{B} \) is even, and \( \vec{d} \cdot \vec{E} \) is odd under both \( P \) and \( T \).

An EDM implies that both \( P \) and \( T \) are violated.

\[ L_{dm} = \frac{1}{2} \left[ D \bar{\mu} \sigma^{\alpha\beta} \frac{1 + \gamma_5}{2} + D^* \bar{\mu} \sigma^{\alpha\beta} \frac{1 - \gamma_5}{2} \right] \mu F_{\alpha\beta} \]

with

\[ \text{Re } D = a_\mu \frac{e}{2m_\mu} \quad \text{and} \quad \text{Im } D = d_\mu \]
Theoretical Value for \((g - 2)\)

- **Electron**: To the level of the experimental error, \(\pm 4\) ppb

\[
\alpha_e(\text{Standard Model}) = \alpha_e(\text{QED with } \gamma, \ e)
\]

Contribution of virtual \(\mu, \tau\), etc. is \(\leq 4\) ppb.
**Theoretical Value for \((g - 2)\)**

- **Electron:** To the level of the experimental error, \(\pm 4\) ppb

\[
a_e(\text{Standard Model}) = a_e(\text{QED with } \gamma, e)
\]

Contribution of virtual \(\mu, \tau, \text{ etc.}\) is \(\leq 4\) ppb.

- **Muon:** The Relative Contribution of heavier things:

\[
\sim \left(\frac{m_\mu}{m_e}\right)^2 \sim 40,000
\]
Theory for Muon \((g - 2)\)

\[
a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{hadronic}) + a_\mu(\text{weak})
\]

\[
a_\mu(\text{New Physics}) = a_\mu(\text{Measured}) - a_\mu(\text{SM})
\]
Theory for Muon \((g - 2)\)

\[
\begin{align*}
\text{QED} & \quad \mu \gamma \gamma + \mu e^- e^+ \gamma \\
& \quad 11,658,470.57 \pm 29 \times 10^{-10} \\

\text{Had} & \quad \mu \gamma h + \mu e^- e^+ h \\
& \quad 692.4 \pm 62 \times 10^{-10} \\

\text{Weak} & \quad \mu \gamma W + \mu \nu W \\
& \quad +38.9 \\

\text{1st + 2nd Order Weak} & = -15.1(4) \times 10^{-10}
\end{align*}
\]
$a_{\mu}(\text{Had})$ from Dispersion Theory

\[ a_{\mu}(\text{had}; 1) = \left( \frac{\alpha m_{\mu}}{3\pi} \right)^2 \int_{4m_{\pi}^2}^{\infty} \frac{ds}{s^2} K(s) R(s) \]

where \[ R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \]
Hadronic light-by-light term

- What’s your sign?
Hadronic light-by-light term

- What's your sign?

- Theorists beware!

\[ \text{X}^{-/+,?} \pm \text{or} \pm \text{?} \]
Hadronic light-by-light term

- What’s your sign?

- Theorists beware!

\[ ? -/+? \quad 8.5 (3.2) \times 10^{-10} \]
Hadronic light-by-light term

- What’s your sign?
  - ⚬ ☽ ☽ ☽
  - ⚬ ☽ ☽ ☽

- Theorists beware!

- It’s + !!!

\[ ?-/+? \quad 8.5 \times 10^{-10} \]
New Physics Contribution?

- substructure?

\[ \Delta a_\mu = \frac{m_\mu^2}{\Lambda^2} \]

\[ \Lambda \geq 5 \text{ Tev} \]
New Physics Contribution?

• substructure?

\[ \Delta a_\mu = \frac{m_\mu^2}{\Lambda^2} \]

\[ \Lambda \geq 5 \text{ Tev} \]

• anomalous gauge boson coupling?

\[ g_w = 2 \ ? \]

W boson substructure?
Supersymmetry

- $\alpha_\mu$ is sensitive to SUSY with large $\tan \beta$

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Supersymmetry

- $a_\mu$ is sensitive to SUSY with large $\tan \beta$
- with equal $\tilde{m}$ and large $\tan \beta$:

$$a_\mu^{(\text{SUSY})} \sim 150 \times 10^{-11} \tan \beta \left(\frac{100 \text{ GeV}}{\tilde{m}}\right)^2$$

$$\sim 1.31 \text{ ppm} \tan \beta \left(\frac{100 \text{ GeV}}{\tilde{m}}\right)^2$$
Connection Between MDM, EDM and $\mu \to e$ in SUSY

In SUSY the MDM, EDM and muon conversion are all inter-related:

$$
\begin{align*}
\mu &\to e \\
\tilde{\mu} &\to \tilde{e} \\
\end{align*}
$$

$$
\begin{pmatrix}
\Delta m_{\tilde{e}\tilde{e}}^2 \\
\Delta m_{\tilde{\mu}\tilde{e}}^2 \\
\Delta m_{\tilde{\tau}\tilde{e}}^2
\end{pmatrix}
\begin{pmatrix}
\Delta m_{\tilde{e}\tilde{\mu}}^2 \\
\Delta m_{\tilde{\mu}\tilde{\mu}}^2 \\
\Delta m_{\tilde{\tau}\tilde{\tau}}^2
\end{pmatrix}
\begin{pmatrix}
m_{\tilde{e}\tilde{e}}^2 \\
m_{\tilde{\mu}\tilde{\mu}}^2 \\
m_{\tilde{\tau}\tilde{\tau}}^2
\end{pmatrix}
$$
BNL AGS E821:

A New Precision Measurement of the Muon \((g - 2)\) Value
at the level of 0.35 ppm

Boston University, Brookhaven National Laboratory, Budker Institute of Nuclear Physics - Novosibirsk, Cornell University, KEK, KVI and Rijksuniversiteit - Groningen, University of Heidelberg, University of Illinois, University of Minnesota, Tokyo Institute of Technology, Yale University
E821 Collaboration (4/02)

The Experimental Technique

Protons from AGS → Target

Pions $\pi^+ \rightarrow \mu^+ \nu_\mu$

Inflector

Storage Ring

Injection Orbit

Ideal Orbit

Kicker Modules

$\mu$
The $\pi$ Production Target
π Decay

The Pion Rest Frame

ν_μ  →  π^+  ←  μ^+

"Forward" muons are highly polarized.
The $\pi$ Decay Channel
Experimental Technique, ctd.

$\mu^+ \nu_\mu$ Inflector

Injection Orbit

Ideal Orbit

Storage Ring

Kicker Modules
**Inflector Geometry**

- Beam
- Inflector
- μ Orbit

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Inflector Exit Geometry

Outer cryostat

\[ \rho = 7112 \text{ mm} \]

upper pole

77 mm

lower magnet pole

heat shield

Muon storage Region 45 mm R
Experimental Signal is $\mu$ Decay

The Muon Rest Frame

Highest energy $e^+$ are along muon spin

The electron carries the muon spin
Spin and Momentum Precession

\[ \omega_C = \frac{eB}{mc\gamma} \quad \omega_S = \frac{geB}{2mc} + (1 - \gamma) \frac{eB}{\gamma mc} \]

\[ \omega (\vec{S} \text{ relative to } \vec{p}) \quad \omega_a = \omega_S - \omega_C = \left( \frac{g - 2}{2} \right) \frac{eB}{mc} \]

The highest energy decay e\(^\pm\) are along the muon spin direction
The need for vertical focusing
The need for vertical focusing

- in (g−2) we store for 4000 turns

helix

not a circle!
The need for vertical focusing

- in (g–2) we store for 4000 turns
- Magnetic focusing conflicts with the need to know $B$ to 0.1 ppm.
The need for vertical focusing

- in \((g−2)\) we store for 4000 turns

- Magnetic focusing conflicts with the need to know \(B\) to 0.1ppm.
- Can we use an electric field?
Spin Motion in $\vec{E}$ and $\vec{B}$-fields:

\[ \vec{\omega}_a = \frac{d\Theta_R}{dt} = \frac{e}{mc} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right] \]

for $\gamma = 29.3$

\[ \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) = 0 \]

Non-relativistic muon in an E field
Electrostatic Quadrupoles
The need for a kick.

\( x_c \approx 77 \text{ mm} \)
\( \beta \approx 10 \text{ mrad} \)
\( B \cdot dl \approx 0.1 \text{ Tm} \)
The Kicker Plates
A Kicker Modulator
The Kicker Current Pulse

Horizontal (radial) Phase Space

Kicker Current vs time (ns)

- Kicker Current (kA)
- Beam

- Ring acceptance: 77mm
- Beam exiting the inflector
- Stored beam
- Ideal kick

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Phase Space \((x, x')\) 1\textsuperscript{st} half turn

ring acceptance
Phase Space \((x, x')\) 2\(^{nd}\) half turn

ring acceptance
Phase Space \((x, x')\) 3\(^{rd}\) half turn

ring acceptance
Phase Space \((x, x')\) 4\(^{th}\) half turn

ring acceptance
Magnetic Circuits

\[ \Phi \oint \frac{dl}{\mu A} = NI \quad \Phi \mathcal{R} = MMF \quad \text{Ohm's law} \]
An array of 17 NMR probes on the trolley maps the B field in the storage region.

366 fixed probes track B with time.
Winding the Superconducting Coils
The Superconducting Coils
Installation of a Pole Piece
The Nude Storage Ring
# Storage Ring Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g-2) Frequency</td>
<td>$f_a \sim 0.23 \times 10^6$ Hz</td>
<td>$\tau_a = 4.37\mu s$</td>
</tr>
<tr>
<td>Muon kinematics</td>
<td>$p_\mu = 3.094$ GeV/c</td>
<td>$\gamma_\mu = 29.3$</td>
</tr>
<tr>
<td></td>
<td>$\gamma = 64.4$ $\mu s$</td>
<td></td>
</tr>
<tr>
<td>Cyclotron Period</td>
<td>$\tau_{cyc} = 149$ ns</td>
<td></td>
</tr>
<tr>
<td>Central Radius</td>
<td>$\rho = 7112$ mm</td>
<td>(280”)</td>
</tr>
<tr>
<td>$B_0 = 1.451$ T</td>
<td>Storage Aperture</td>
<td>9.0 cm circle</td>
</tr>
<tr>
<td>In one lifetime:</td>
<td>432 revolutions around ring</td>
<td></td>
</tr>
<tr>
<td></td>
<td>14.7 (g-2) periods</td>
<td></td>
</tr>
</tbody>
</table>
\[ \langle B \rangle = \int M(r, \theta) B(r, \theta) r \, dr \, r \, d\theta \]

\[ B(r, \theta) = \sum_{n=0}^{\infty} r^n \left[ c_n \cos n\theta + s_n \sin n\theta \right] \]

One slice in azimuth.

Muon Distribution

\[ M(r, \theta) = \sum [\gamma_m(r) \cos m\theta + \sigma_m(r) \sin m\theta] \]
And $< B >_{\phi}$ for 2001 is:
The $e^\pm$ Energy Spectrum

\[ \delta \varepsilon = \frac{\delta \omega_a}{\omega_a} = \frac{\sqrt{2}}{2\pi f_a \tau \mu N^{\frac{1}{2}} A} \]

\[ \int N A^2 = \int_{E_T}^{E_{max}} N(E) A^2(E) dE \]
The Detector Geometry

- Muon momentum
- Muon spin
- Sci-Fi Calorimeter module
  Measures Energy and time
- Spin forward, more high energy e
- Spin backward, less high energy e
- 400 MHz digitizer

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The Scalloped Vacuum Chamber
Time Spectrum, $\sim 4 \times 10^9 e^-$, $E > 1.8\text{GeV}$, $\sigma_{\text{stat}} \approx 0.7 \text{ppm}$

$$f(t) = N_0 e^{-\lambda t} \left[1 + A \cos(\omega_a t + \phi)\right]$$

![Electron time spectrum (2001)](image-url)
The Muon Distribution

The distribution of equilibrium radii

Measured distribution and Monte Carlo agree.
\omega_a = \frac{e}{m} \alpha \mu \langle B \rangle

Data analysis done Blind

- \langle \omega_p \rangle
  - 2 independent analyses for \omega_p
- \langle \omega_a \rangle
  - 5 independent analyses for \omega_a
ωa Fitting Function

Nature gives us 5 parameters:

\[ f(t) = N_0 e^{-\lambda t} [1 + A \cos(\omega_a t + \phi)] \]

Storage ring plus bunched beam gives more:

Fourier Transform of the residuals from a fit to 5 parameters
Coherent Betatron Oscillations

\[ f_{\text{CBO}} = f_C - f_x = (1 - \sqrt{1 - n}) f_C \]

(\( \lambda_{\text{CBO}} \sim 14 \) turns)

\( f_{\text{CBO}} \) amplitude modulates the \( e^\pm \) signal.
Modified Time Distribution

\[ N_p = N_0 e^{-\frac{t}{\tau}} (1 + A' \sin (\omega_a t + \phi')) \times (1 + A_{CBO}(t) \cos (\omega_{CBO} t + \phi_{CBO})) \]

- \[ A' = A(1 + A_1(t) \cos (\omega_{CBO} t + \phi_1)) \]
- \[ \phi' = \phi(1 + A_2 \cos (\omega_{CBO} t + \phi_2)) \]
- \[ A_1 \text{ and } A_2 \rightarrow \text{artificial shifts in } \omega_a \text{ up to 4 ppm} \]

in individual detectors when not accounted for.

Shifts largely cancel in sum of detectors due to circular symmetry. Cancellation factor \( \approx 9 \)
\[ \omega_a = \frac{e}{m} a_\mu < B > \]

Remove offsets and divide to determine

\[ R = \frac{\omega_a}{\omega_p} \]

and

\[ a_\mu = \frac{R}{\lambda - R} \quad \text{where} \quad \lambda = \frac{\mu_\mu}{\mu_p} \]
Removing the offsets

- Written reports from each “analyzer”.

1 2 3 4 5

\[ \omega \]
Removing the offsets

- Written reports from each “analyzer”.
- Review committee for $\omega_p$ analysis.
Removing the offsets

- Written reports from each “analyzer”.
- Review committee for $\omega_p$ analysis.
- Review committee for $\omega_a$ analysis.
Removing the offsets

- Written reports from each “analyzer”.
- Review committee for $\omega_p$ analysis.
- Review committee for $\omega_a$ analysis.
- Written reports from review committees
Removing the offsets

- Written reports from each “analyzer”.
- Review committee for $\omega_p$ analysis.
- Review committee for $\omega_a$ analysis.
- Written reports from review committees
- Full collaboration agrees at a meeting to accept the reports.
Removing the offsets

- Written reports from each “analyzer”.
- Review committee for $\omega_p$ analysis.
- Review committee for $\omega_a$ analysis.
- Written reports from review committees
- Full collaboration agrees at a meeting to accept the reports.
To open the box?
To open the box?
\[ \omega_a = \frac{e}{m} a_\mu < B > \]

Remove offsets and divide to determine

\[ R = \frac{\omega_a}{\omega_p}, \quad a_\mu = \frac{R}{\lambda - R}, \quad \lambda = \frac{\mu_\mu}{\mu_p} \]

Add corrections for radial \( \vec{E} \)-field and vertical "pitching motion". \((+0.77 \pm 0.06 \text{ ppm})\)

\[ a_{\mu^-} = 11 659 214(8)(3) \times 10^{-10} \quad (\pm 0.7 \text{ ppm}) \]

\[ \sigma_{Stat} = 0.66 \text{ ppm}, \quad \sigma_{Syst-\omega_p} = 0.17 \text{ ppm}, \quad \sigma_{Syst-\omega_a} = 0.21 \text{ ppm}. \]
Where we came from:

(10 ppm) ~1983

CERN $\mu^+$

CERN $\mu^-$

$116,590,000 \times 10^{-11}$

Theory

$a_\mu$
Where we are today:

- CERN $\mu^+$
- CERN $\mu^-$
- E821 (97) $\mu^+$
- E821 (98) $\mu^+$
- E821 (99) $\mu^+$
- E821 (00) $\mu^+$
- E821 (01) $\mu^-$

- (13 ppm)
- (10 ppm)
- (9.4 ppm)
- (5 ppm)
- (1.3 ppm)
- (0.7 ppm)
- (0.7 ppm)

$\mu = 116.592000 \times 10^{-11}$

$\alpha_\mu = 116.595000 \times 10^{-11}$

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Only $e^+e^-$:

- (10 ppm) CERN $\mu^+$
- (9.4 ppm) CERN $\mu^-$
- (13 ppm) E821 (97) $\mu^+$
- (5 ppm) E821 (98) $\mu^+$
- (1.3 ppm) E821 (99) $\mu^+$
- (0.7 ppm) E821 (00) $\mu^+$
- (0.7 ppm) E821 (01) $\mu^-$

$\alpha\mu$:

- 116 590 000
- 116 591 000
- 116 592 000
- 116 593 000
- 116 594 000
- 116 595 000

$\times 10^{-11}$

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E821 Measurements

**Diagram Description**
- The diagram plots $a_\mu \times 10^{-10}$ as a function of $1 - 11659000$.
- It shows measurements for $\mu^+$, $\mu^-$, and the average (Avg.) for both experiment and theory.
- The diagram includes error bars for both experiment and theory data points.

**Data Points**
- **Experiment**:
  - $\mu^+$
  - $\mu^-$
  - Average
- **Theory**:
  - $[\tau]$
  - $[e^+ e^-]$

**Additional Information**
- B. Lee Roberts, University of New Mexico 23 Jan. 2004 – p.74/77
τ for 2 (π)? An Intermezzo

What are the issues which might make the τ-decay data complicated to compare with $e^+ e^-$ data.

- CVC?
- isospin violation?
- experimental problems?
**CVC Tests:** \( \text{Br}(\tau \rightarrow \nu_\tau \pi^- \pi^0) \)

BR comparison with the prediction using CVC, lepton universality and \( e^+ e^- \rightarrow \pi^+ \pi^- \) as input. Discrepancy is 2.9\( \sigma \); \( |F_{\pi}|^2 \) from \( e^+ e^- \) and \( \tau \) differ.

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Stay tuned!